Building Discrete Self Similar Fractals in Seeded TA

Authors: **Ryan Knobel,** Adrian Salinas, Robert Schweller, Timothy Wylie

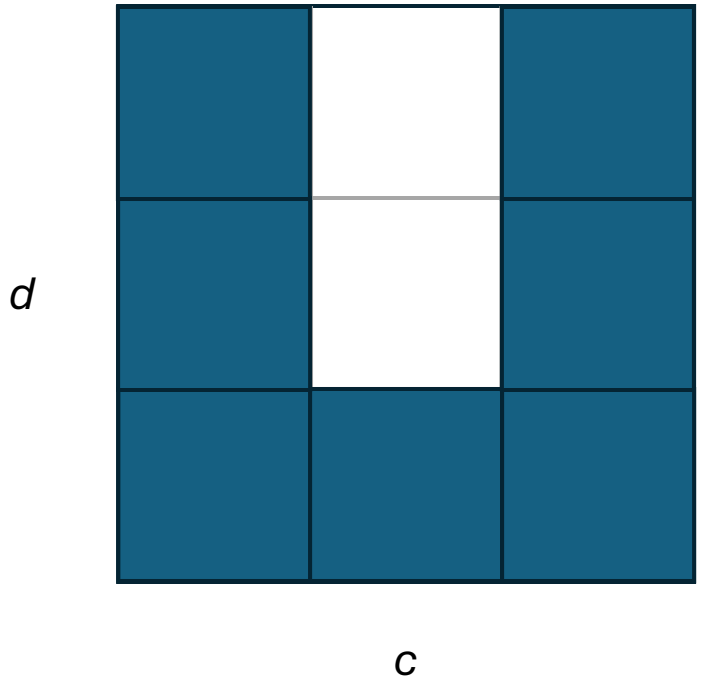
Overview

1. Discrete Self-Similar Fractals
2. Previous Work
3. Seeded Tile Automata
4. Our Construction
5. Future Work

Discrete Self-Similar Fractals

What are they?

Generator



Generator

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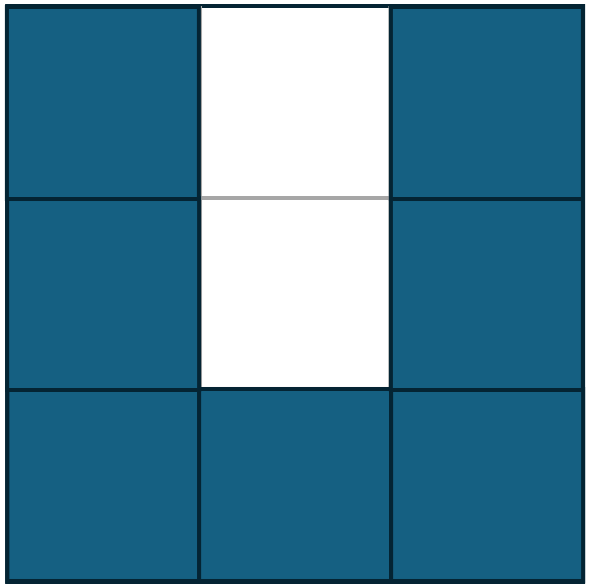

Generator

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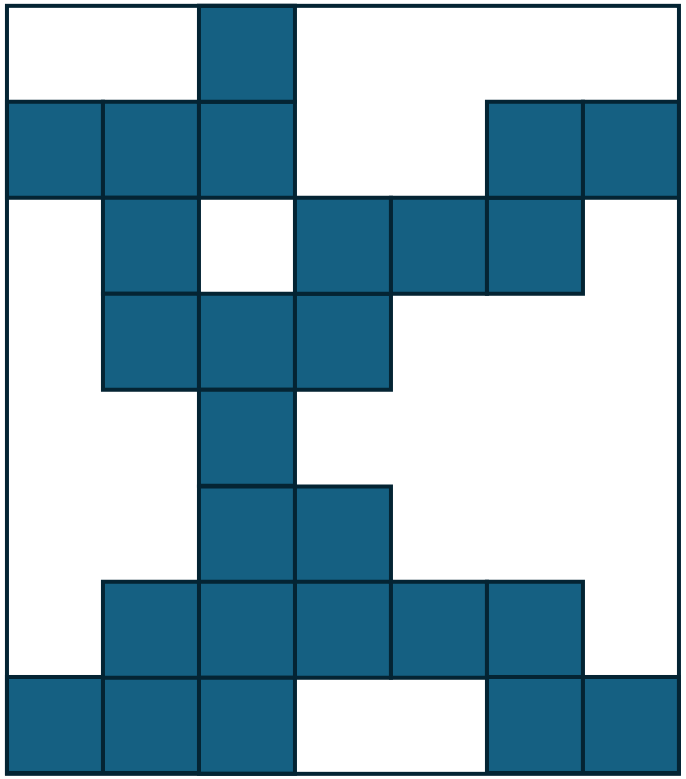
2. Feasible



Generator

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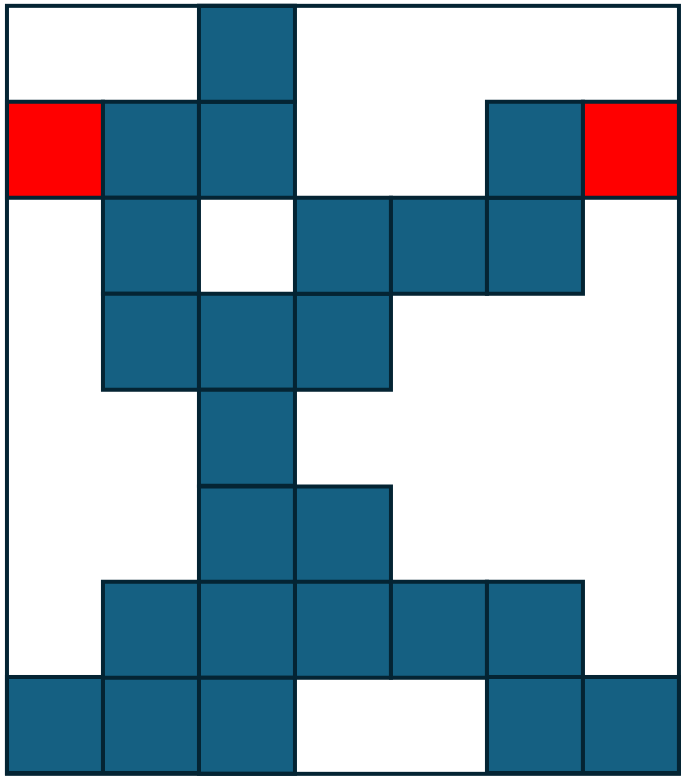

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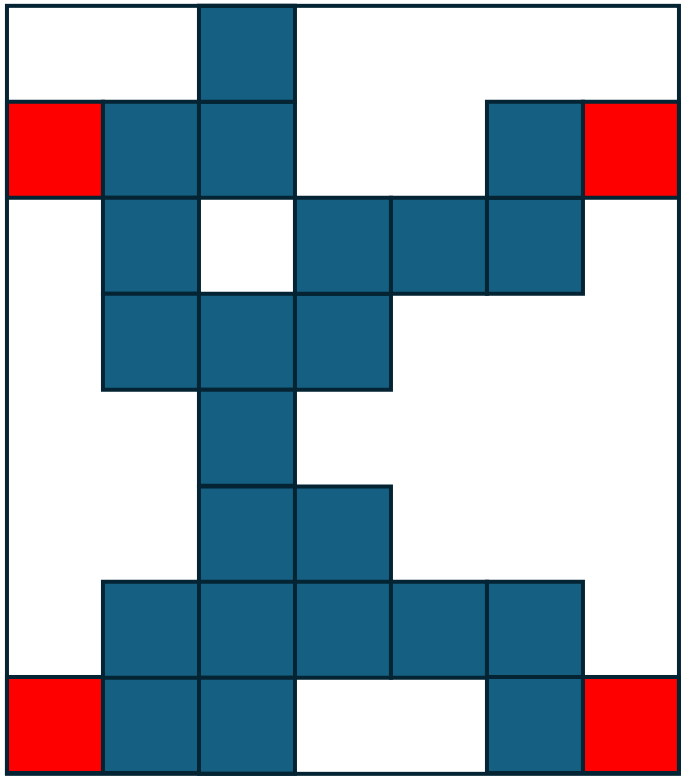

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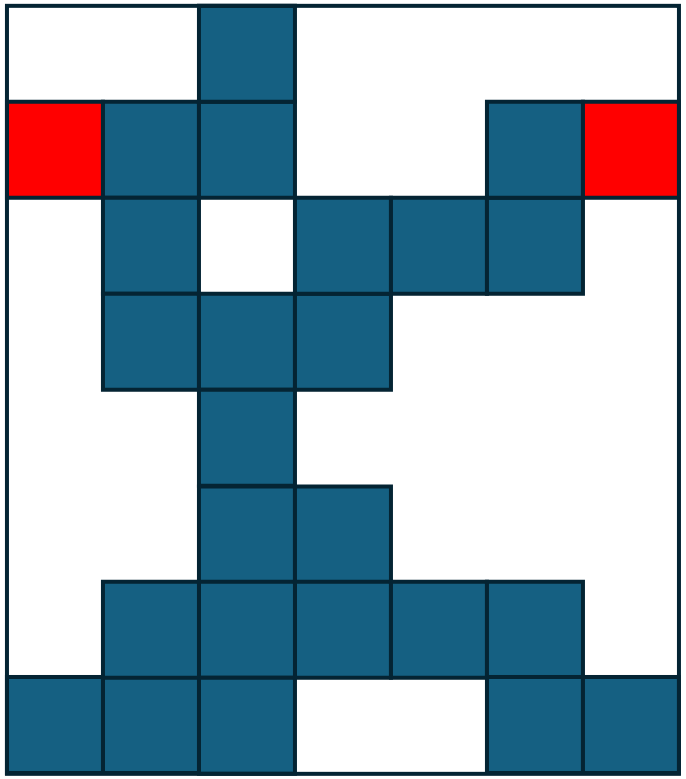

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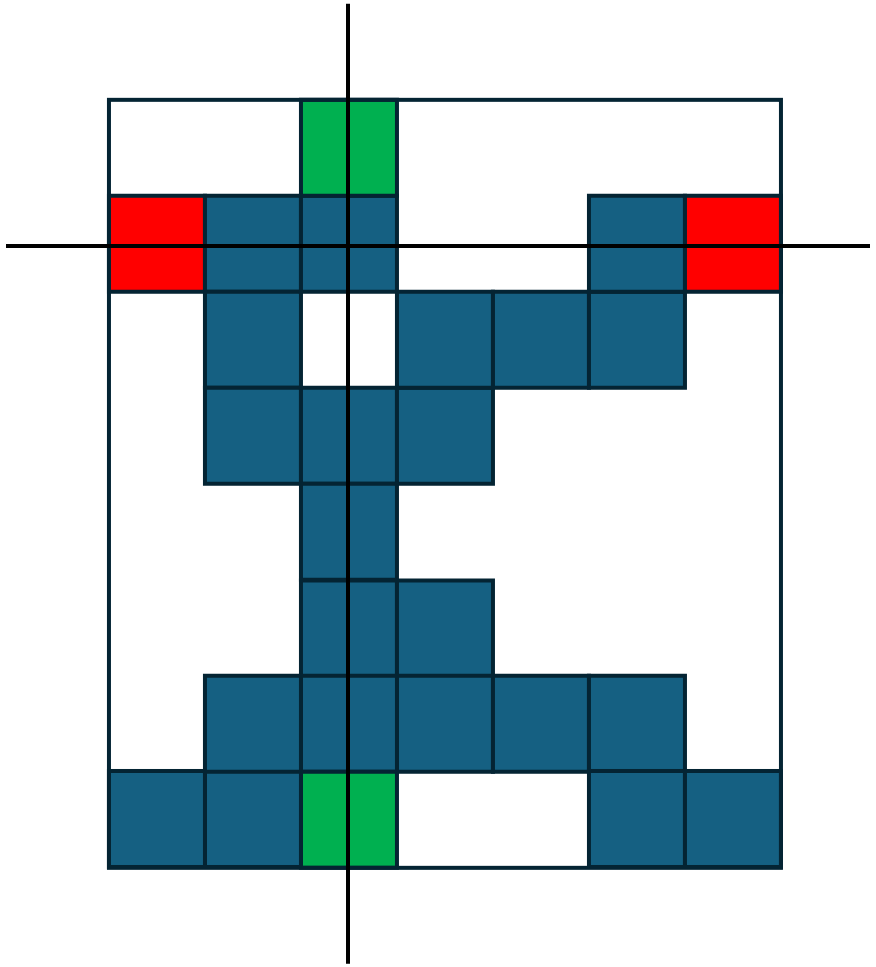

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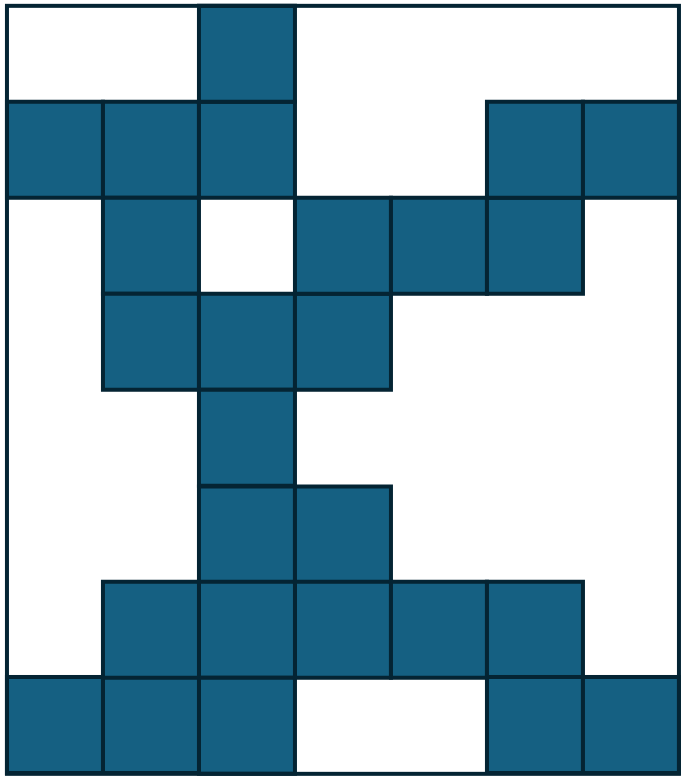

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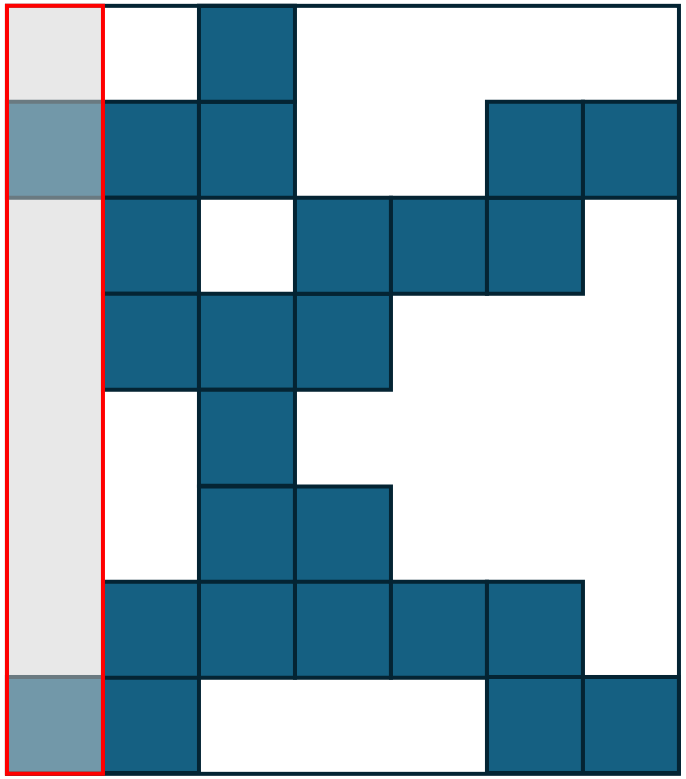

Generator

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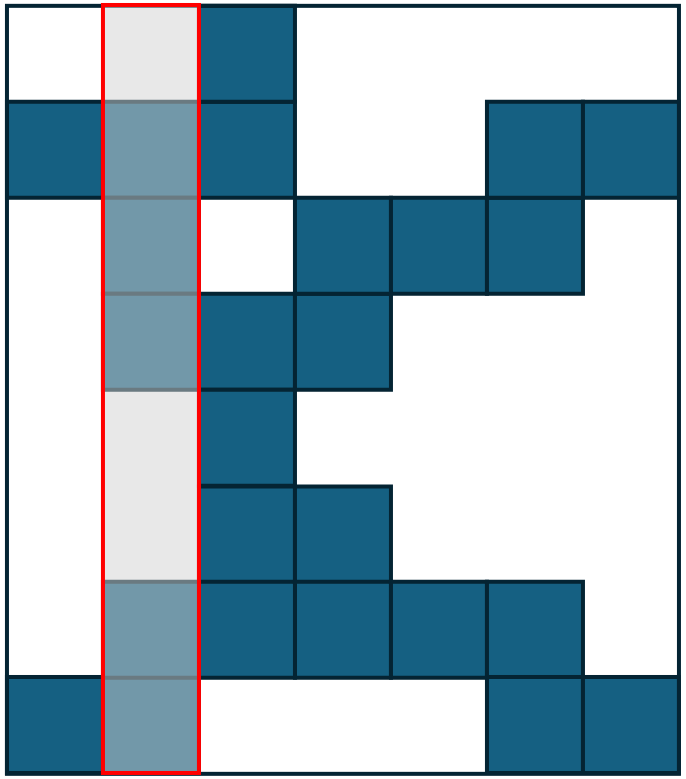

Generator

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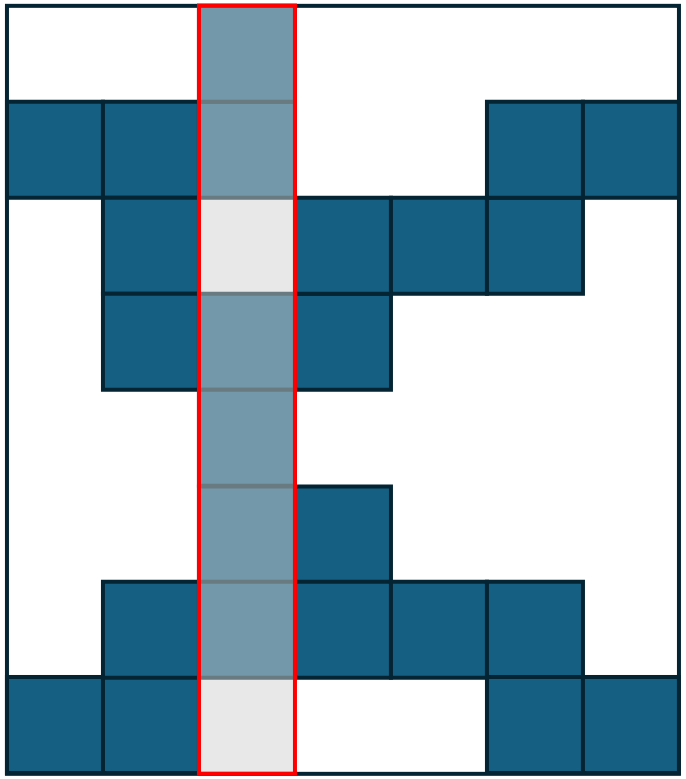

Generator

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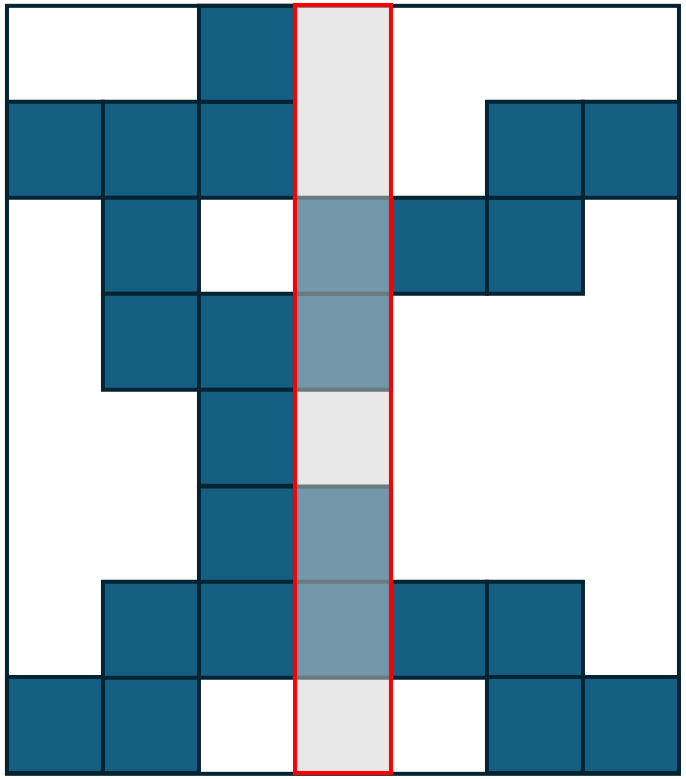

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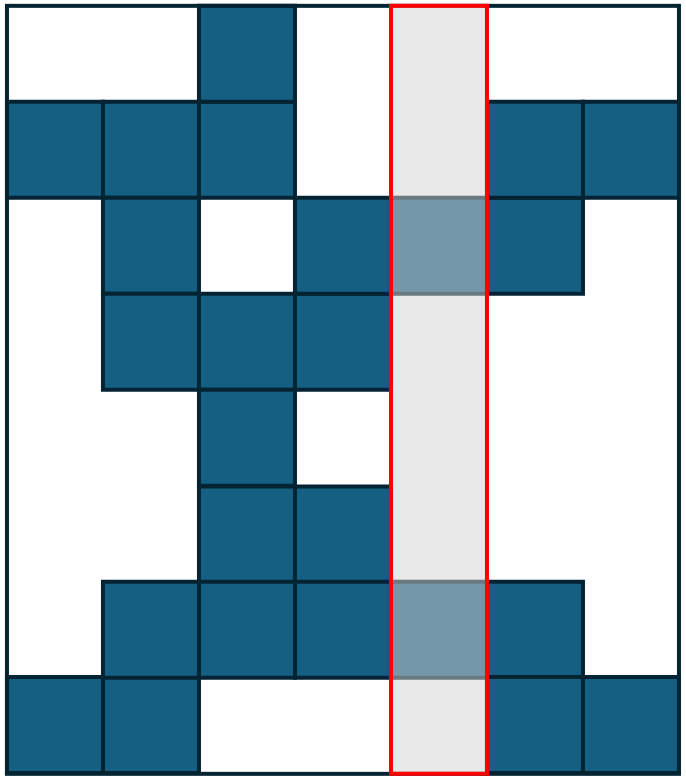

Generator

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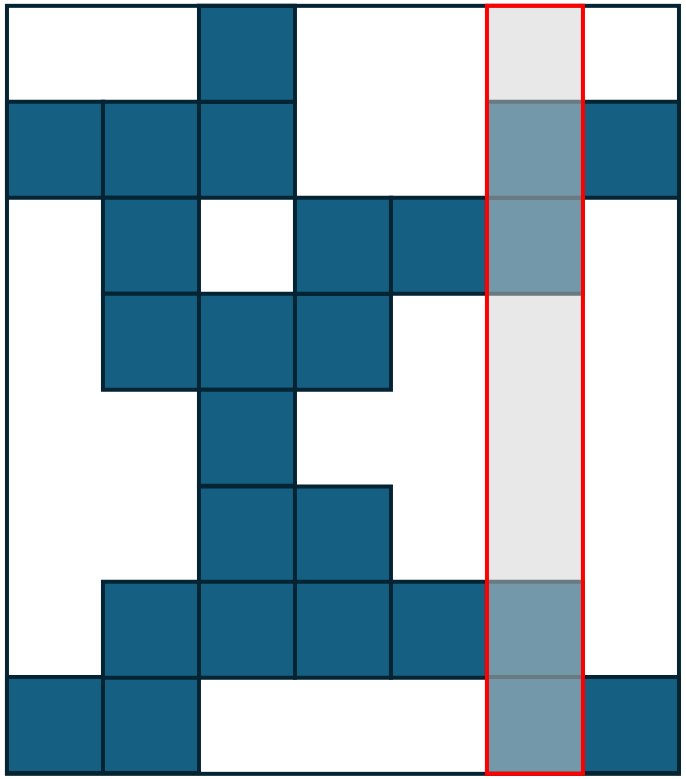

Generator

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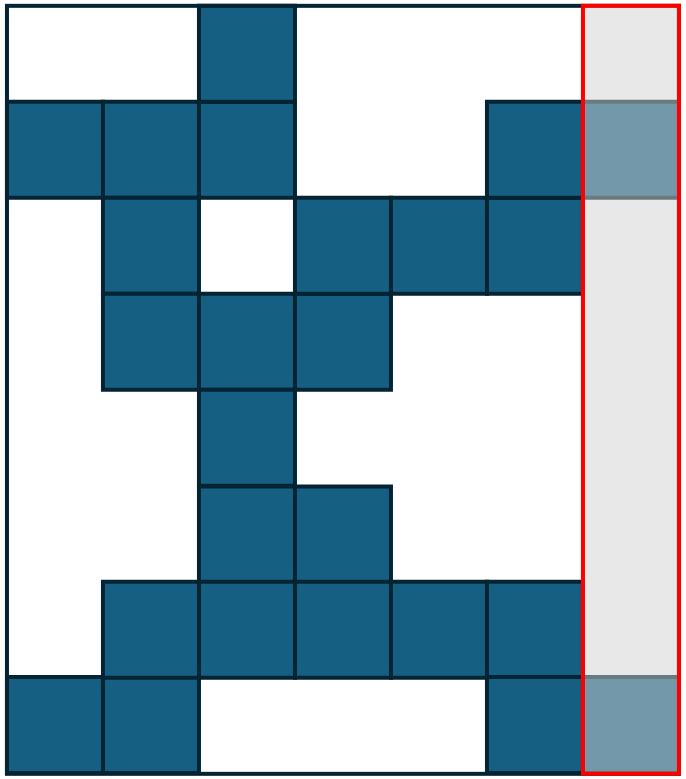
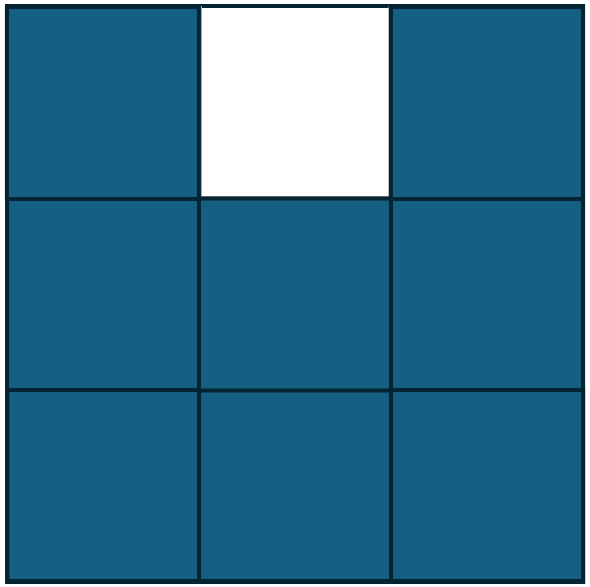
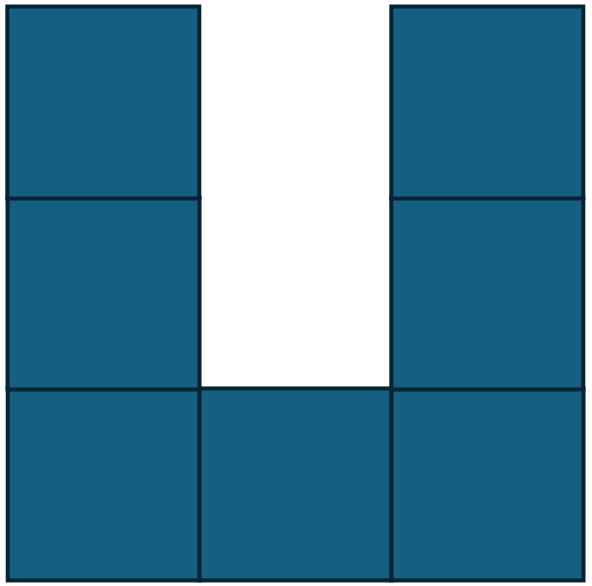


Generator

1. Connected
2. Feasible
3. *Hamiltonian path*

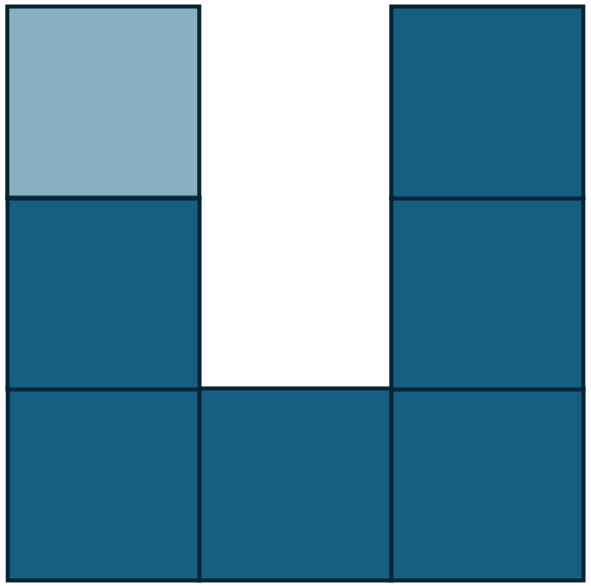


Discrete Self-Similar Fractal

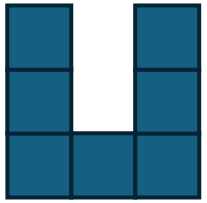


Stage 1

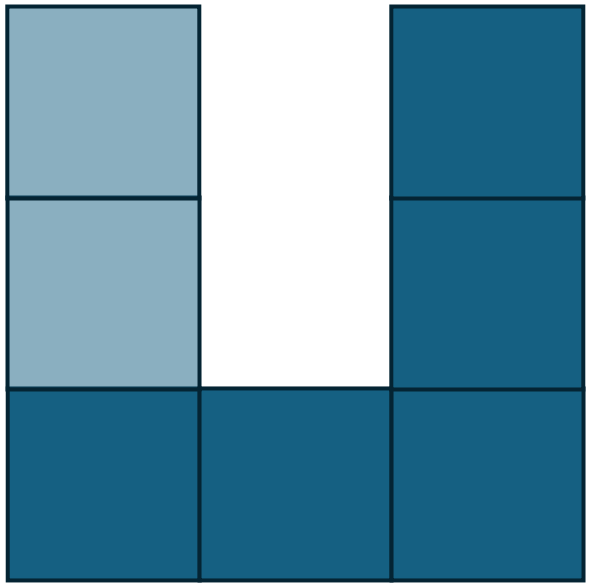
Discrete Self-Similar Fractal



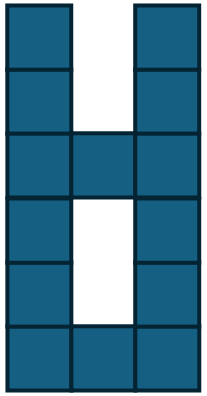
Stage 1



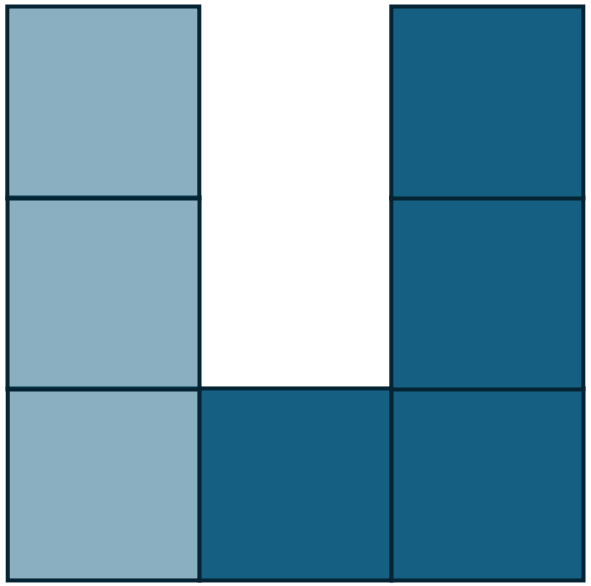
Discrete Self-Similar Fractal



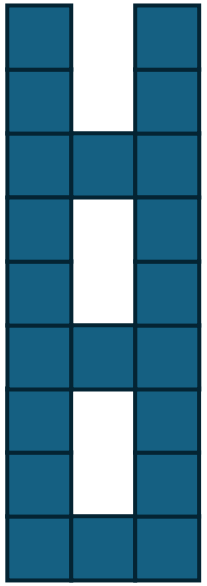
Stage 1



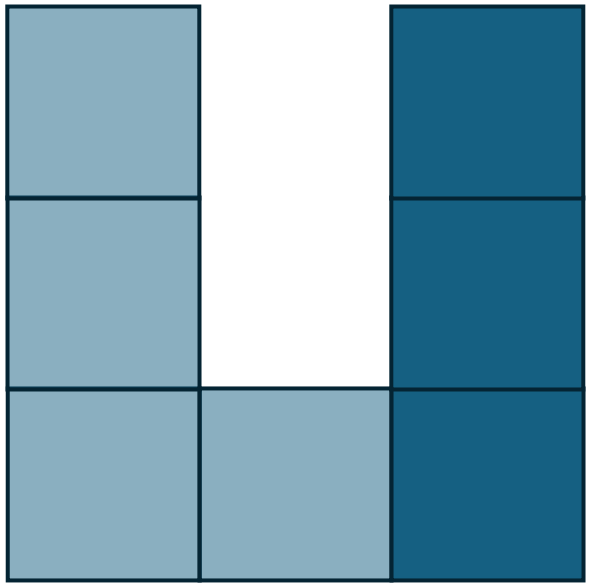
Discrete Self-Similar Fractal



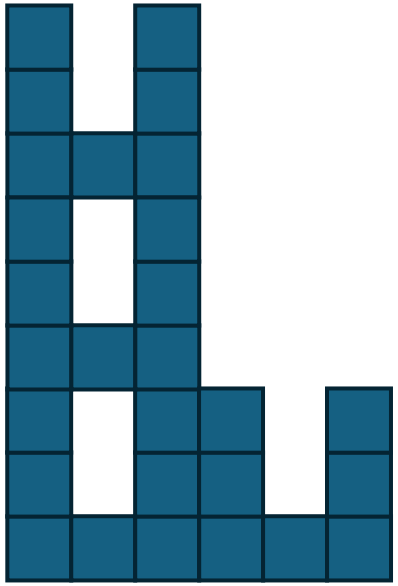
Stage 1



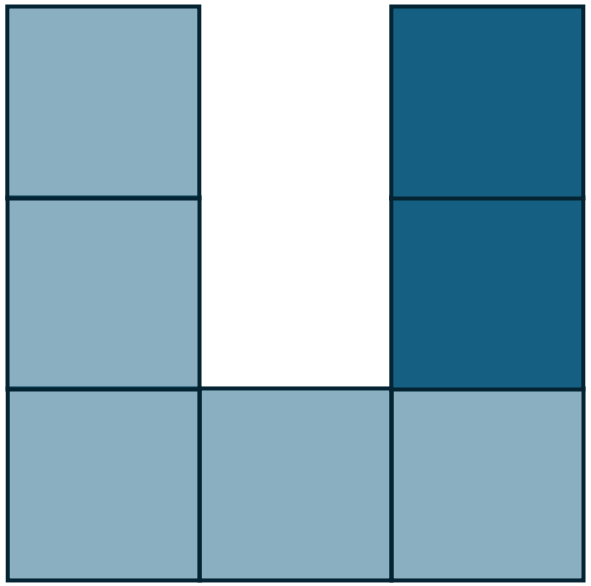
Discrete Self-Similar Fractal



Stage 1



Discrete Self-Similar Fractal



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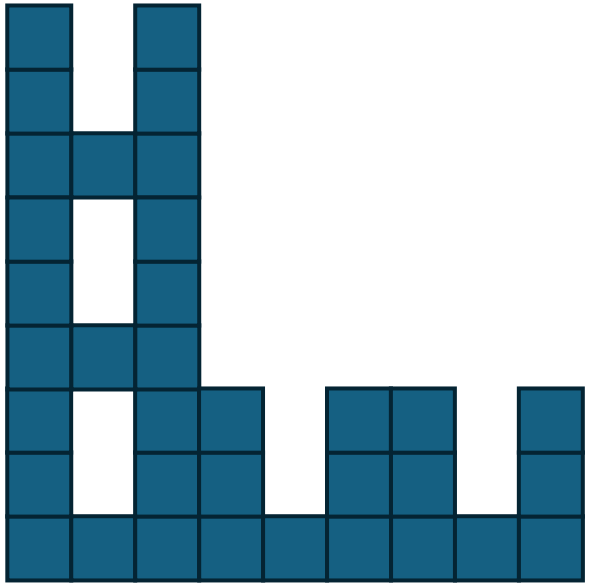
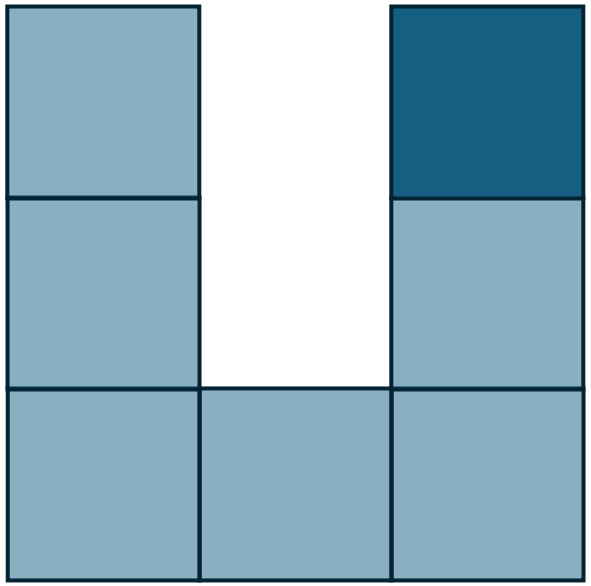
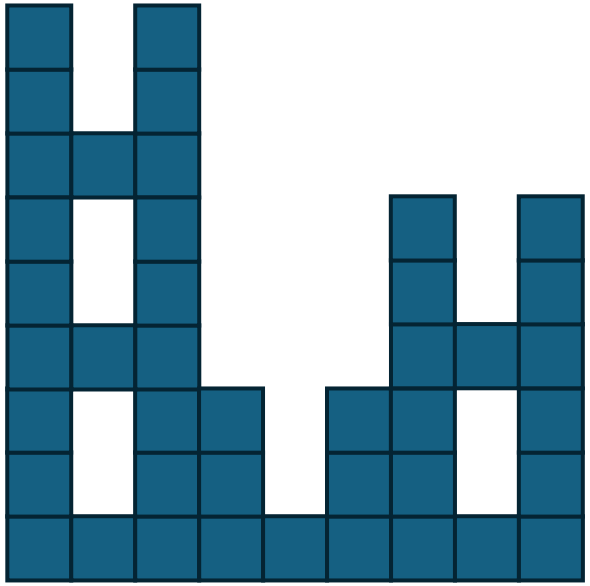


Discrete Self-Similar Fractal



Stage 1



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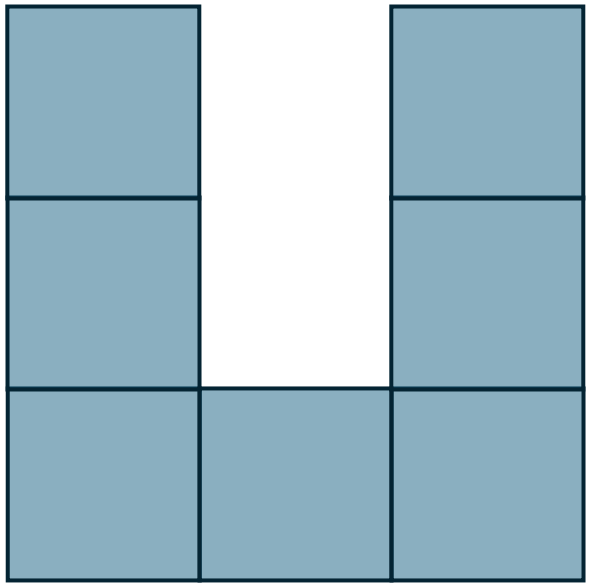
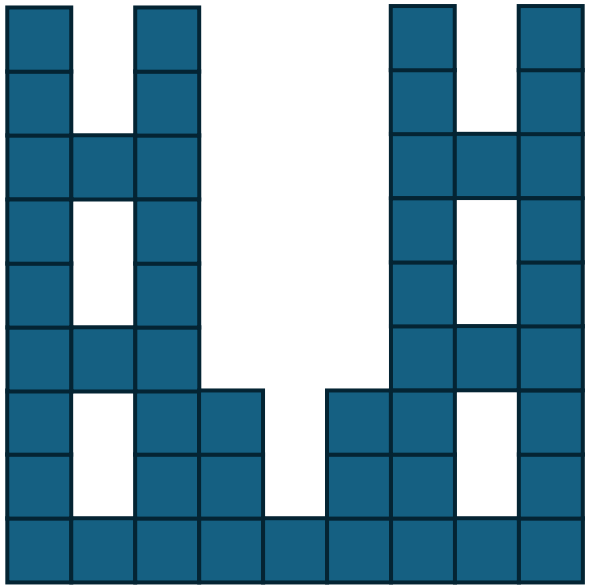
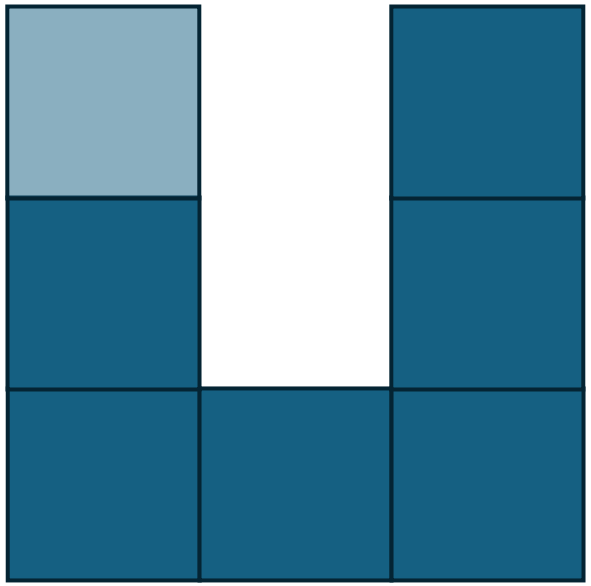


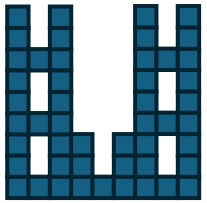
Stage 1



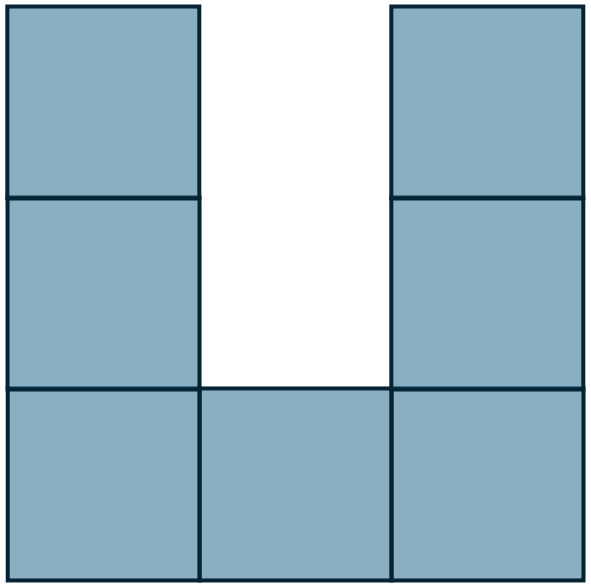
Stage 2

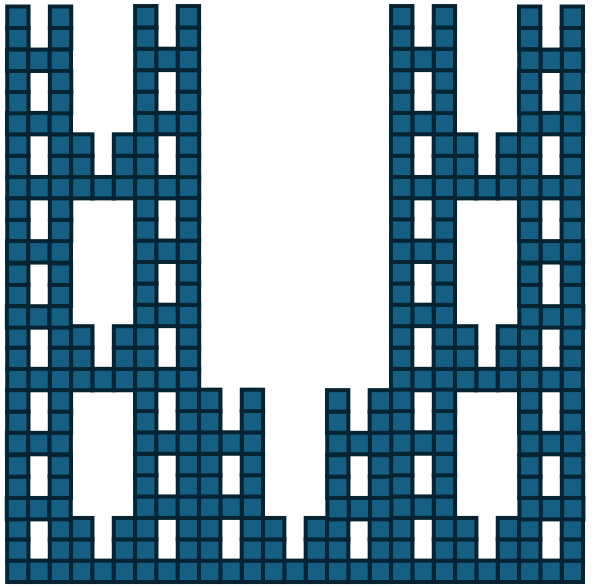
Discrete Self-Similar Fractal





Discrete Self-Similar Fractal

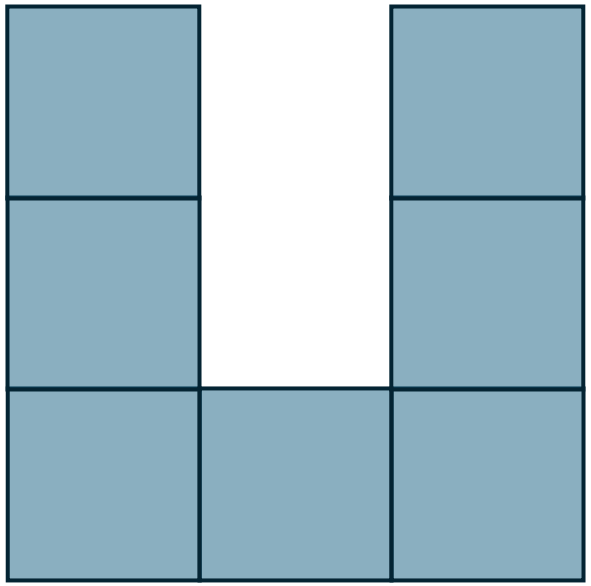


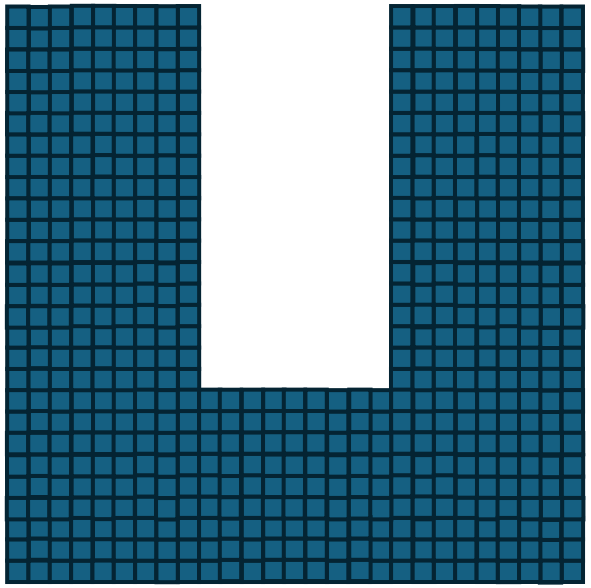


Strict!

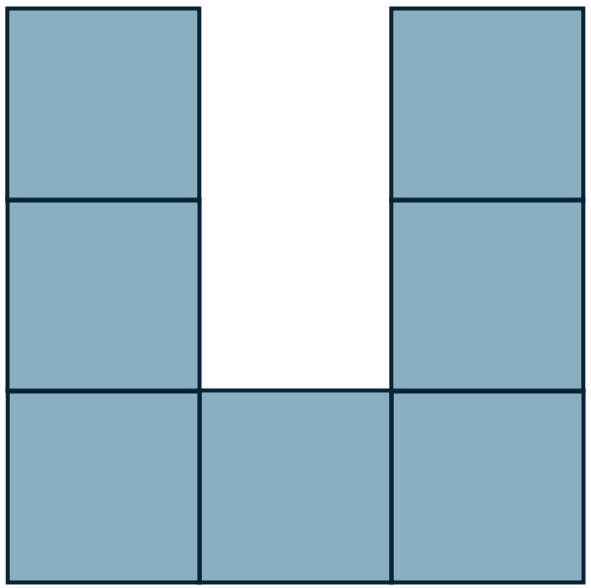
Stage 3

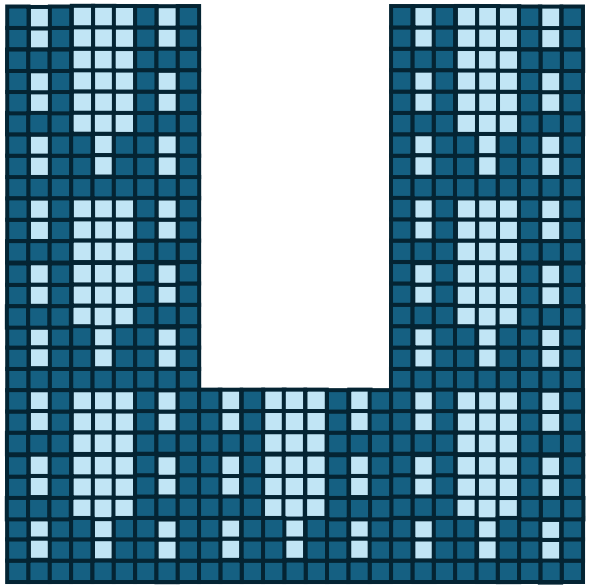
Discrete Self-Similar Fractal





Discrete Self-Similar Fractal





Weak!

Previous Work

Why is this important?

DNA Self-Assembly

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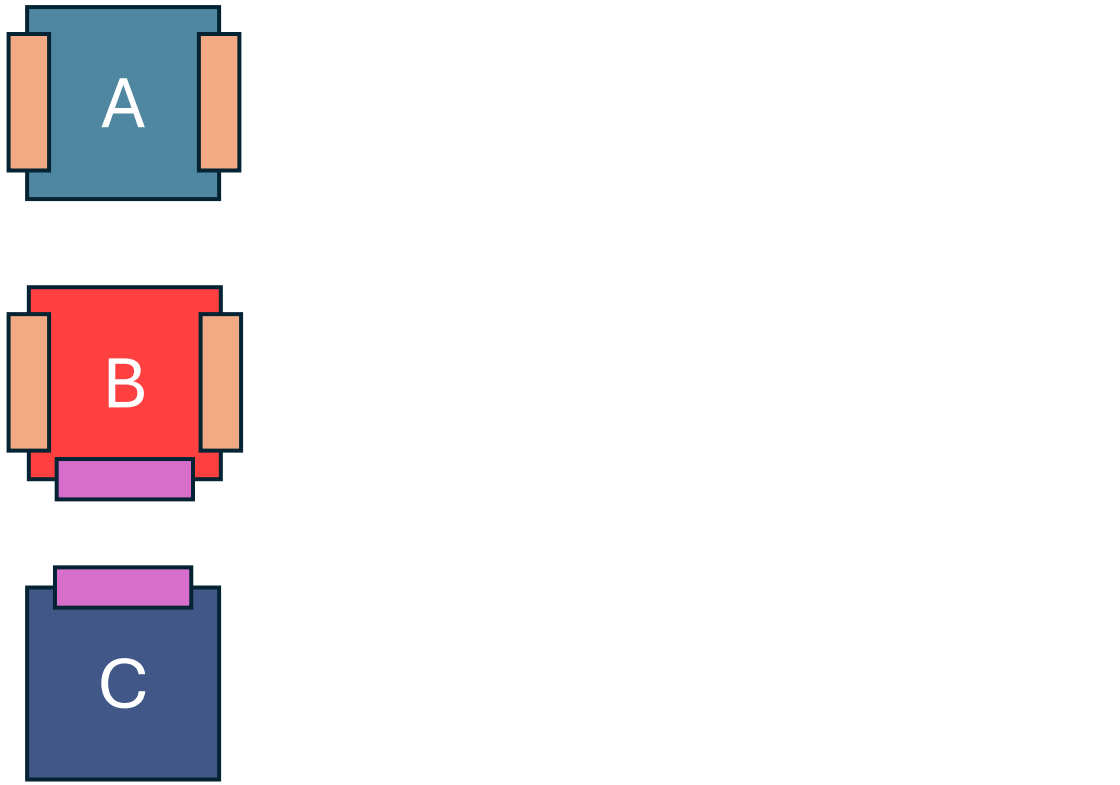

DNA Self-Assembly

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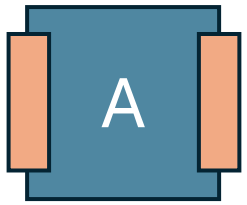

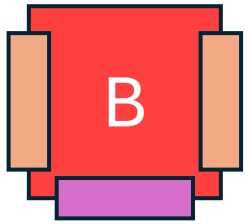

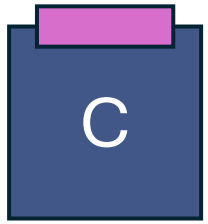

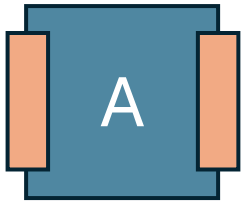

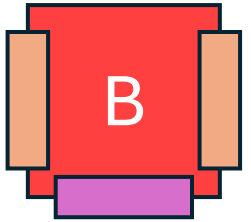

DNA Self-Assembly

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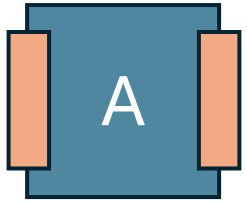

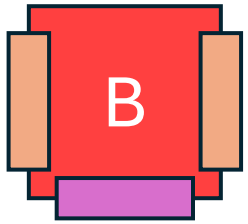

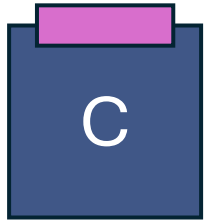

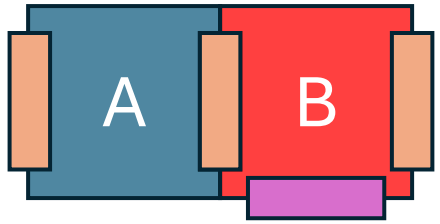

DNA Self-Assembly

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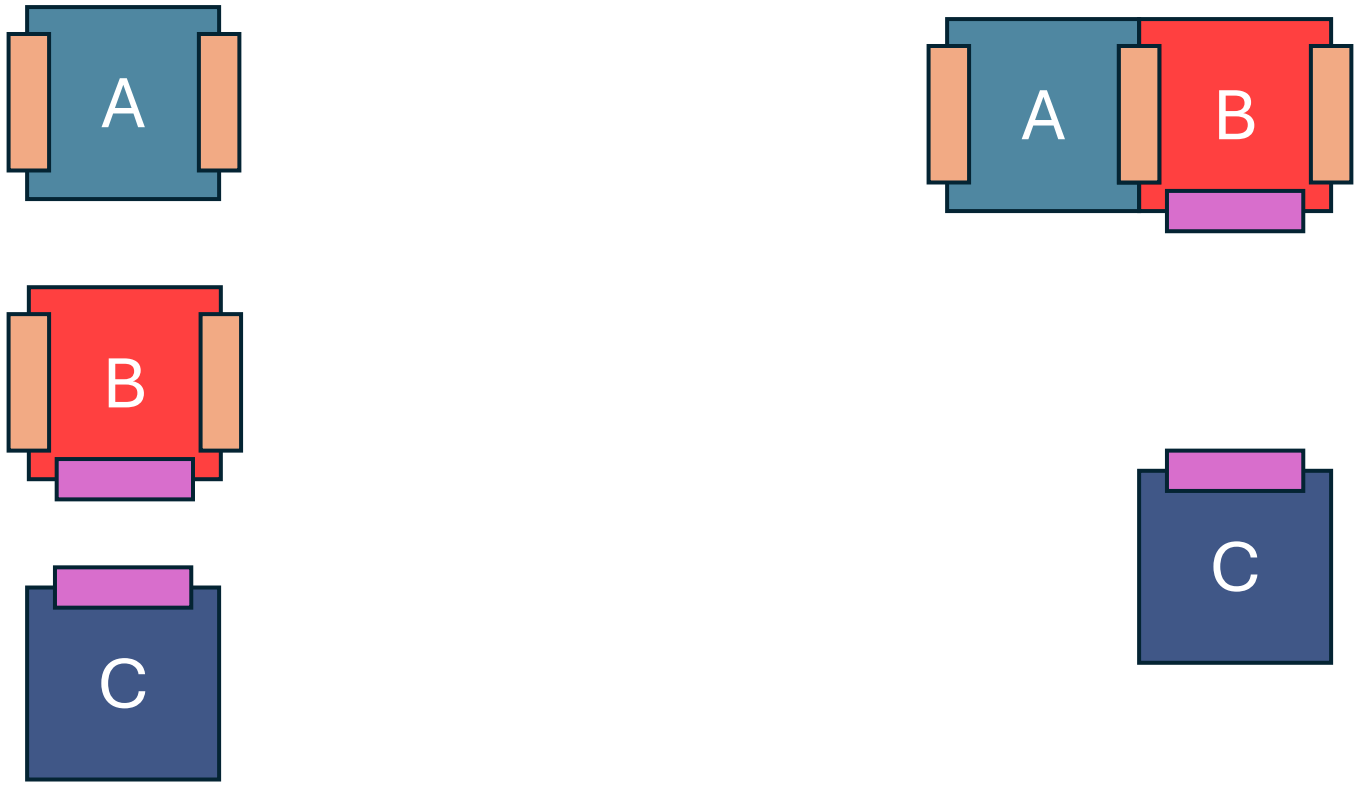

DNA Self-Assembly

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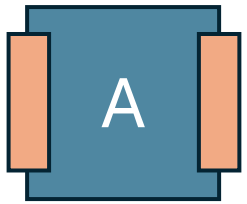
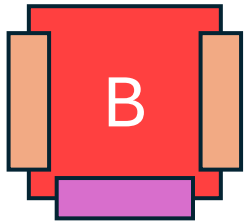
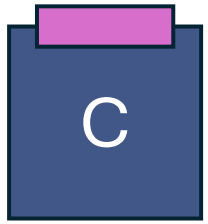
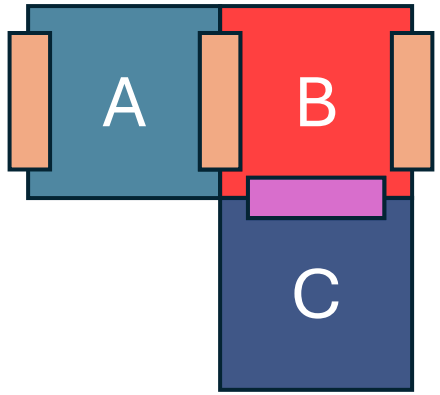




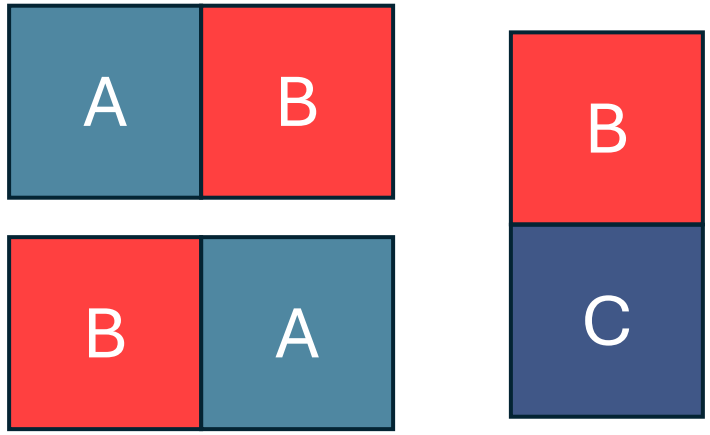




DNA Self-Assembly

Tiles:

Affinities:



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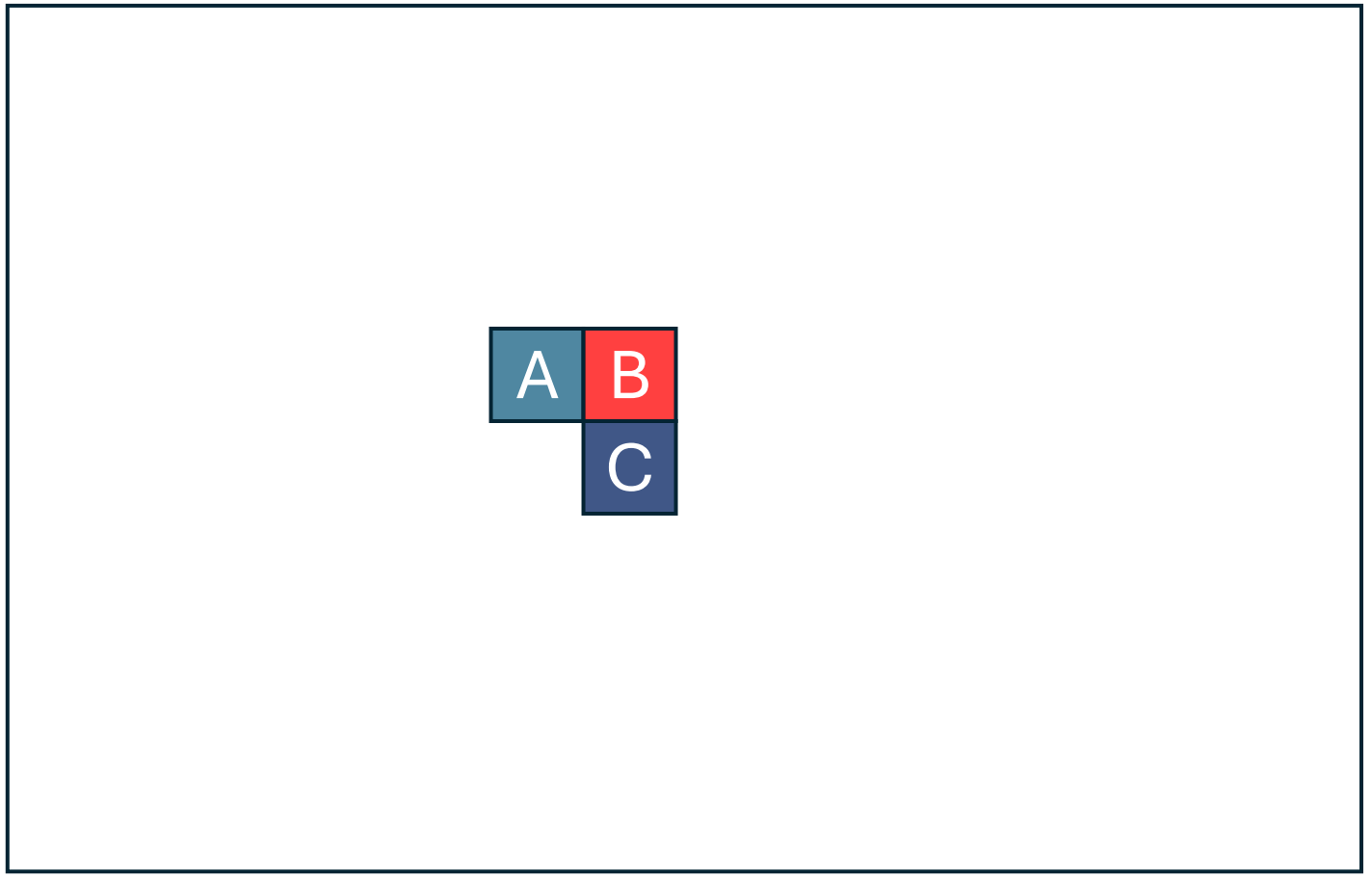
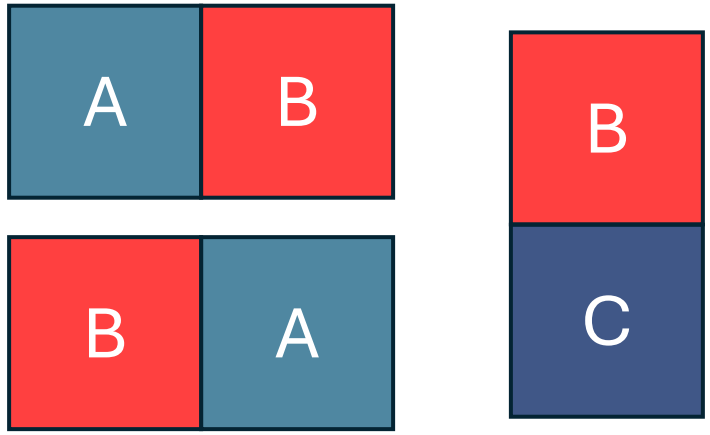


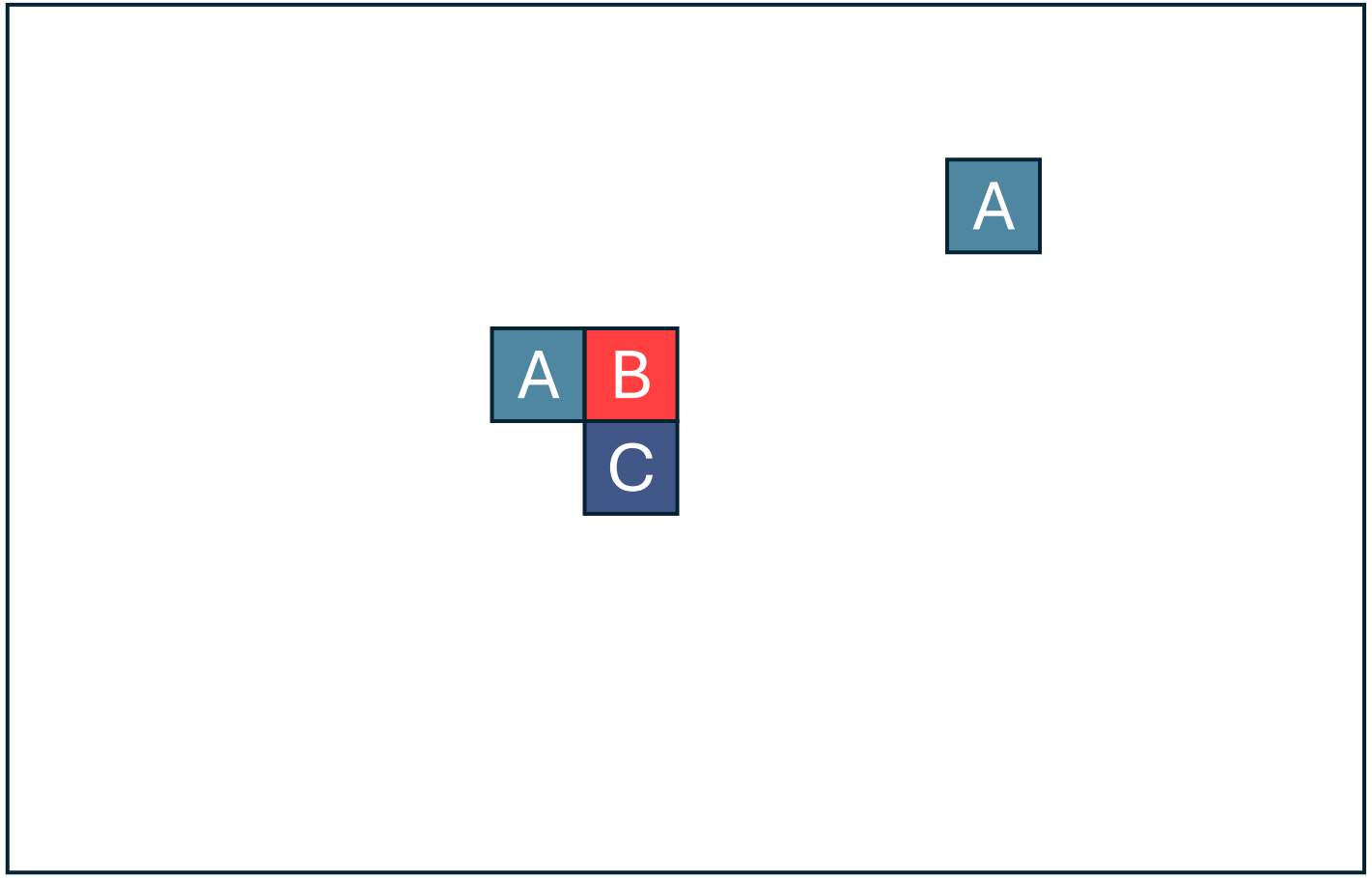
DNA Self-Assembly

Tiles:

Affinities:



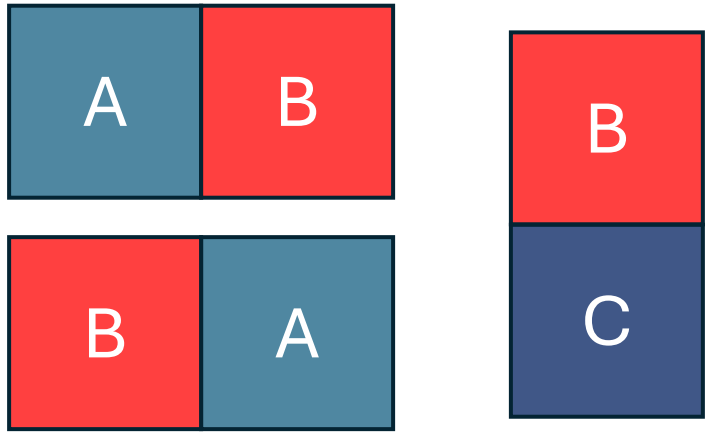
System:



DNA Self-Assembly

Tiles:

Affinities:



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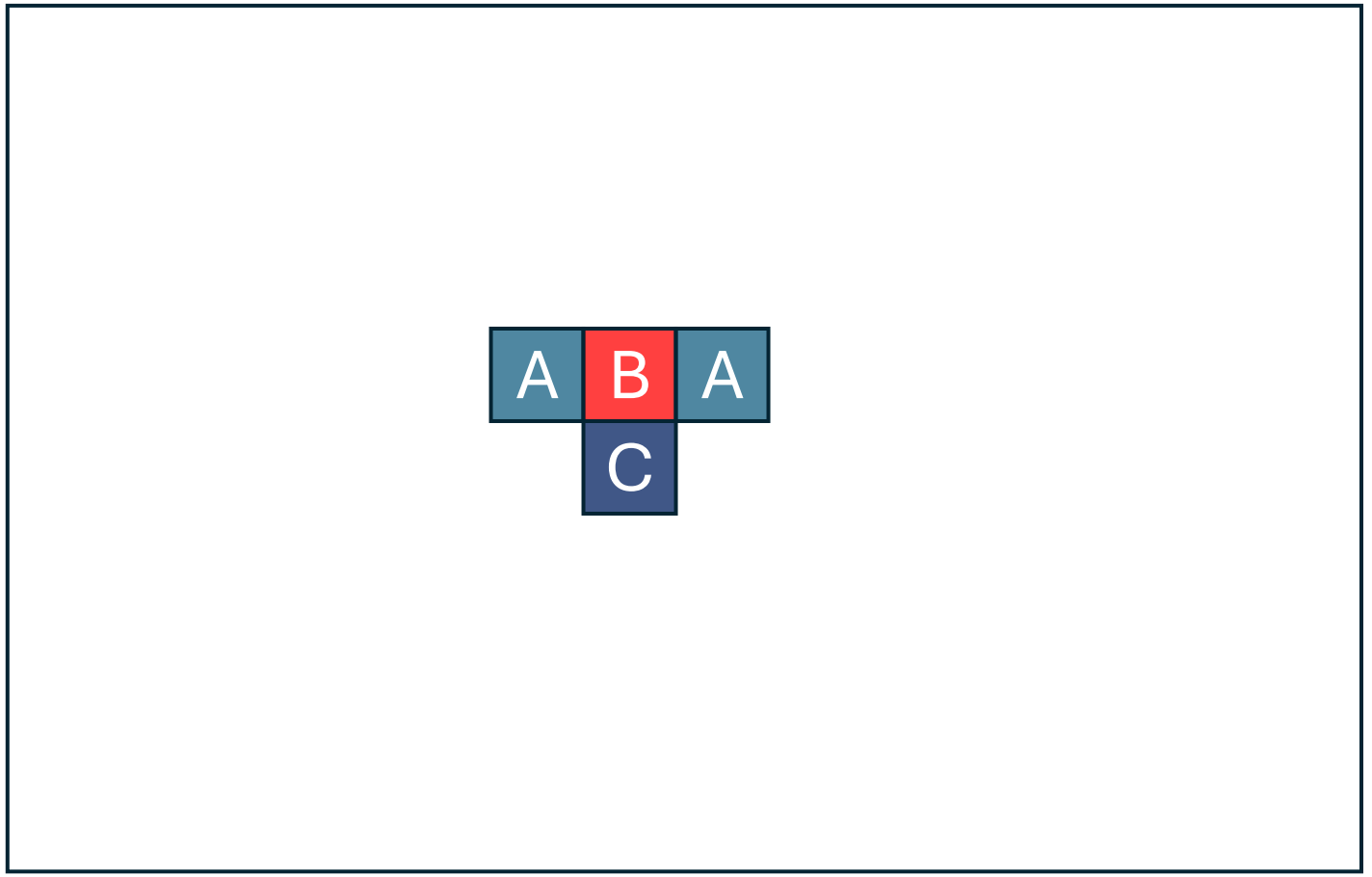
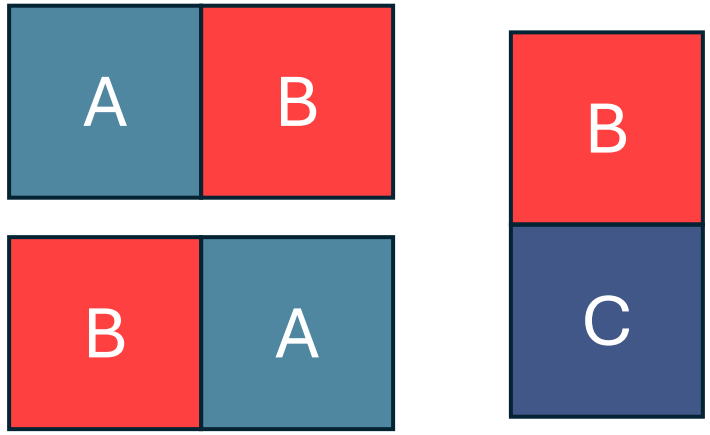


DNA Self-Assembly

Tiles:

Affinities:



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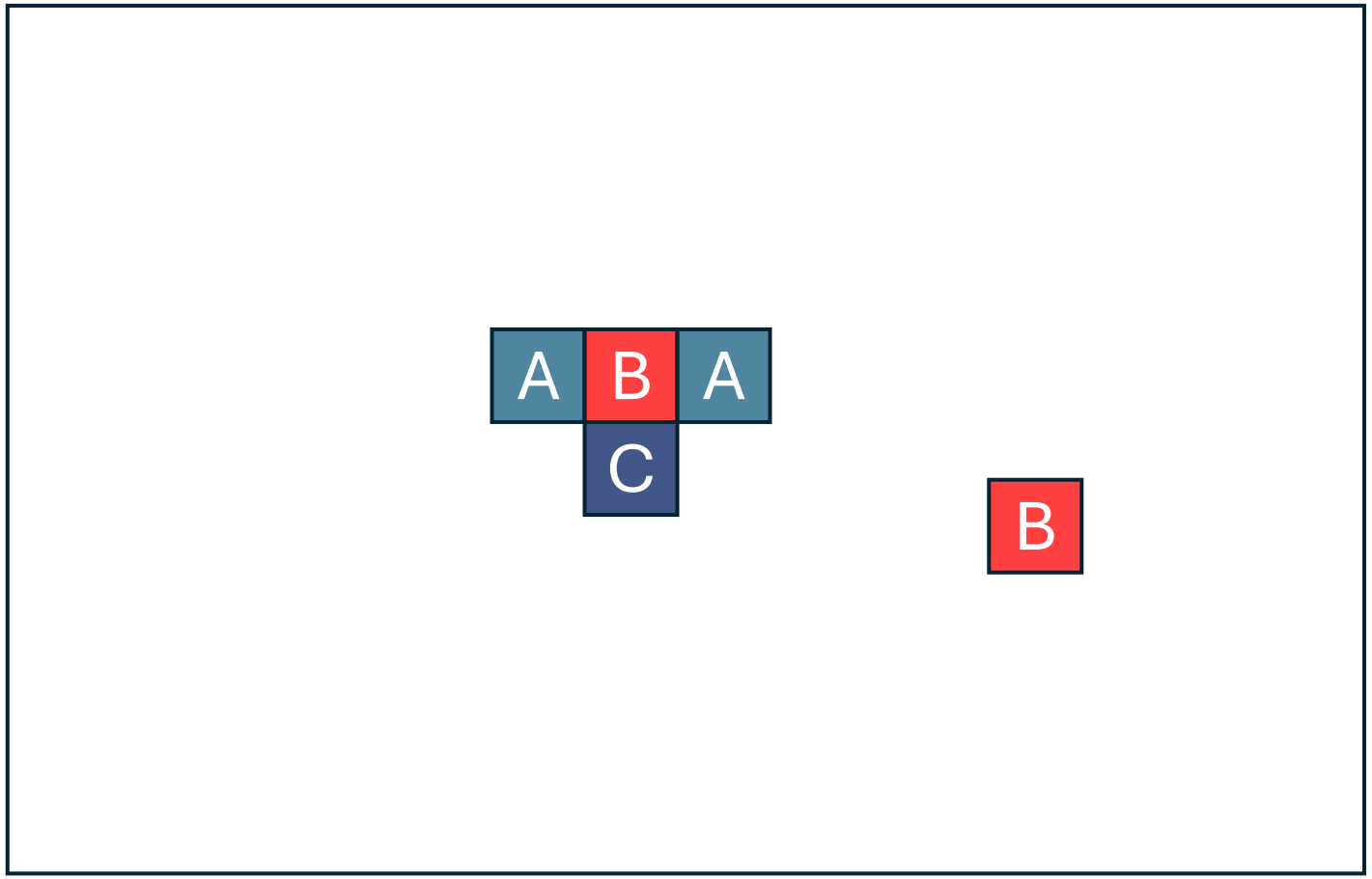
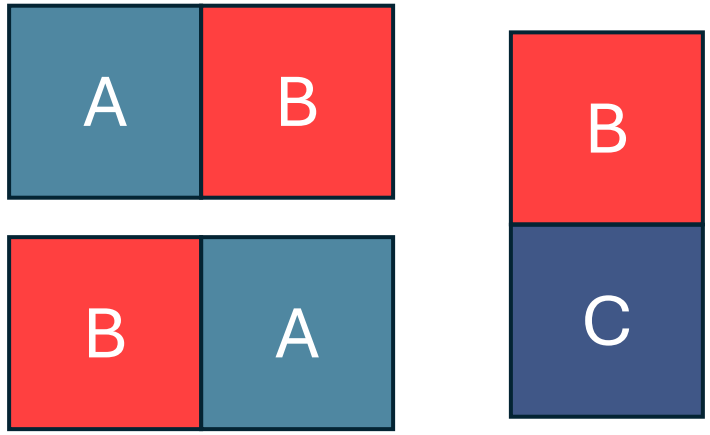


DNA Self-Assembly

Tiles:

Affinities:



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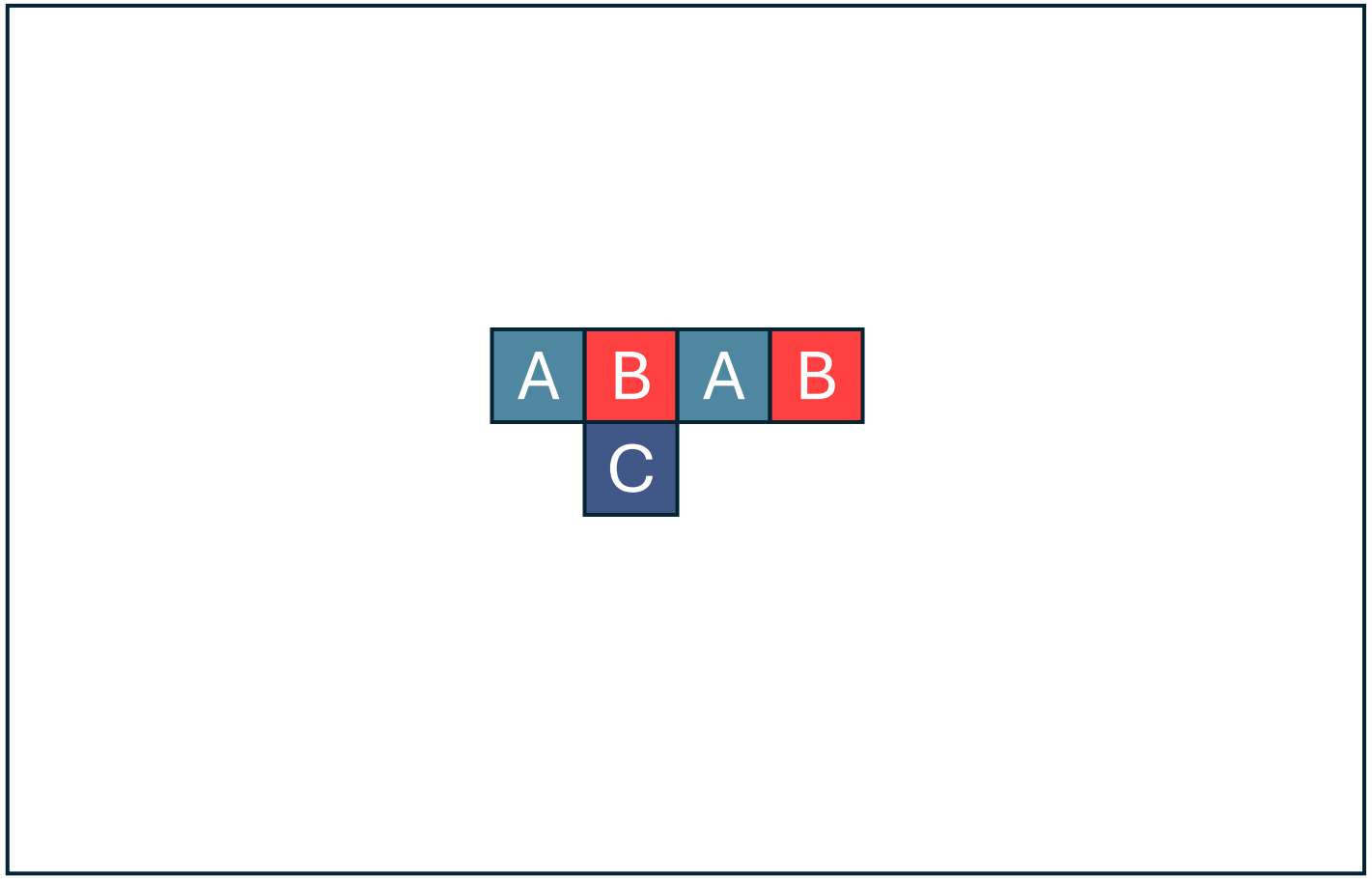
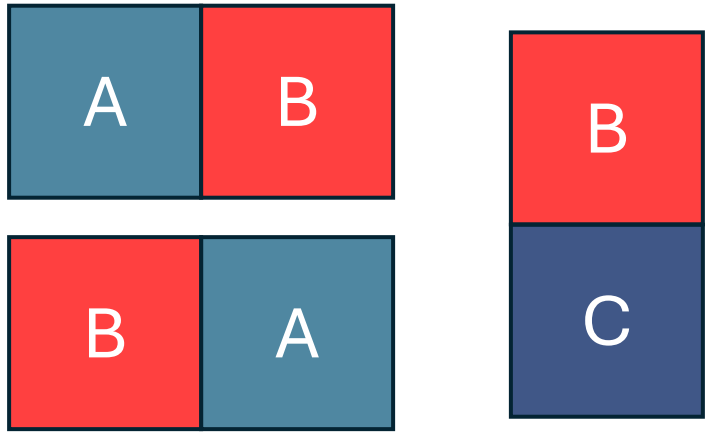


DNA Self-Assembly

Tiles:

Affinities:



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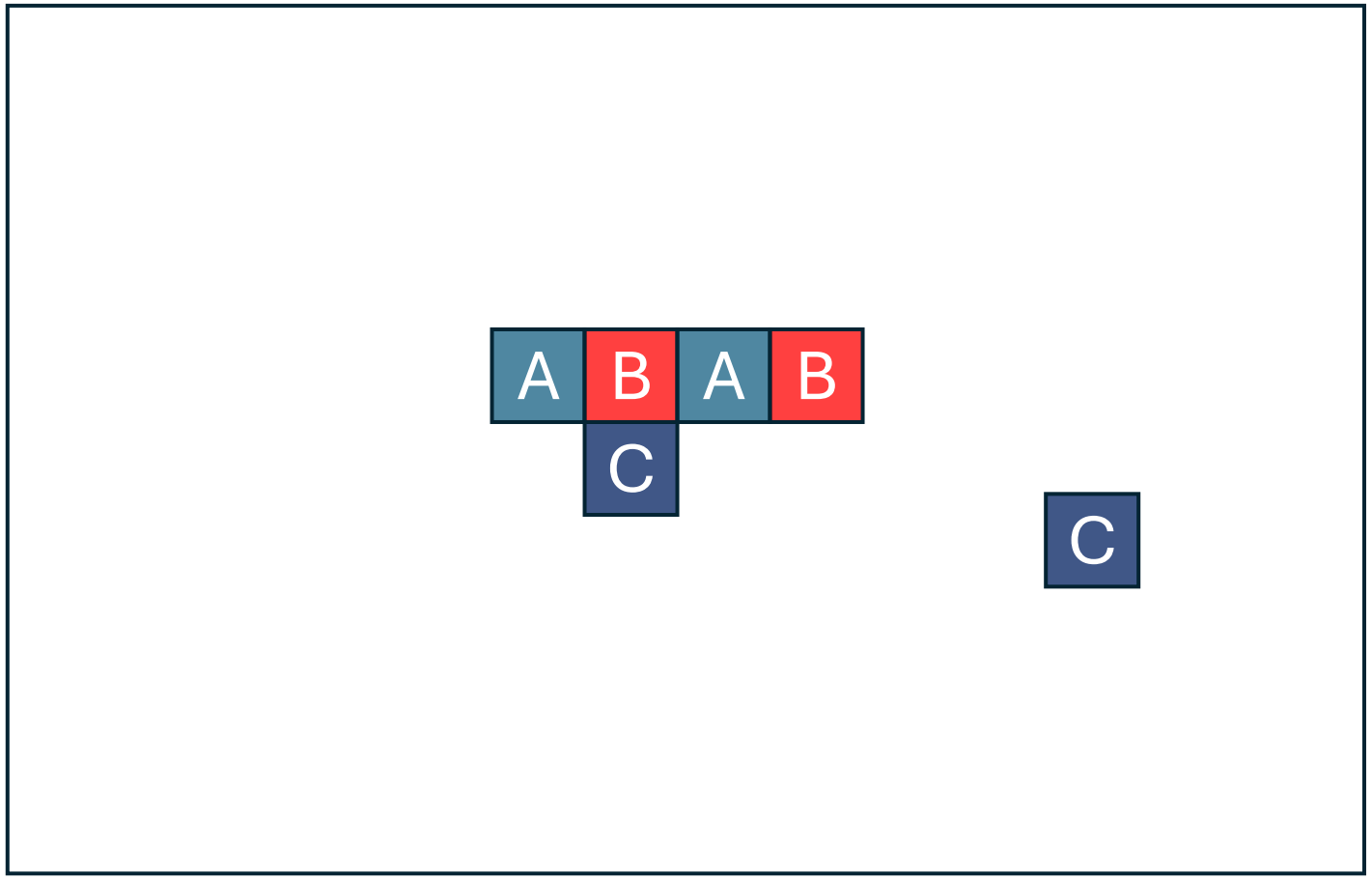

DNA Self-Assembly

Tiles:

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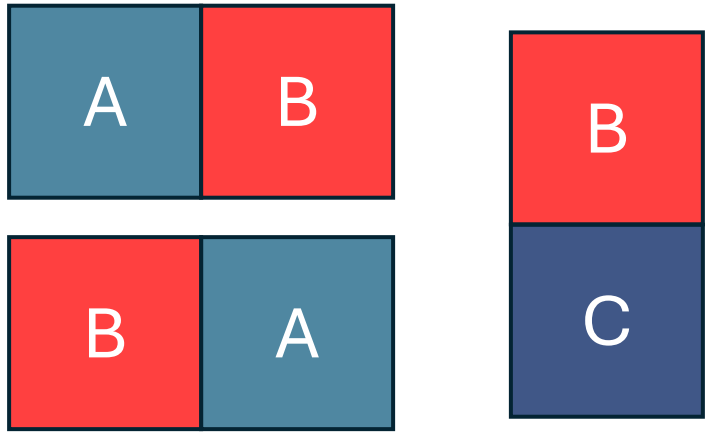

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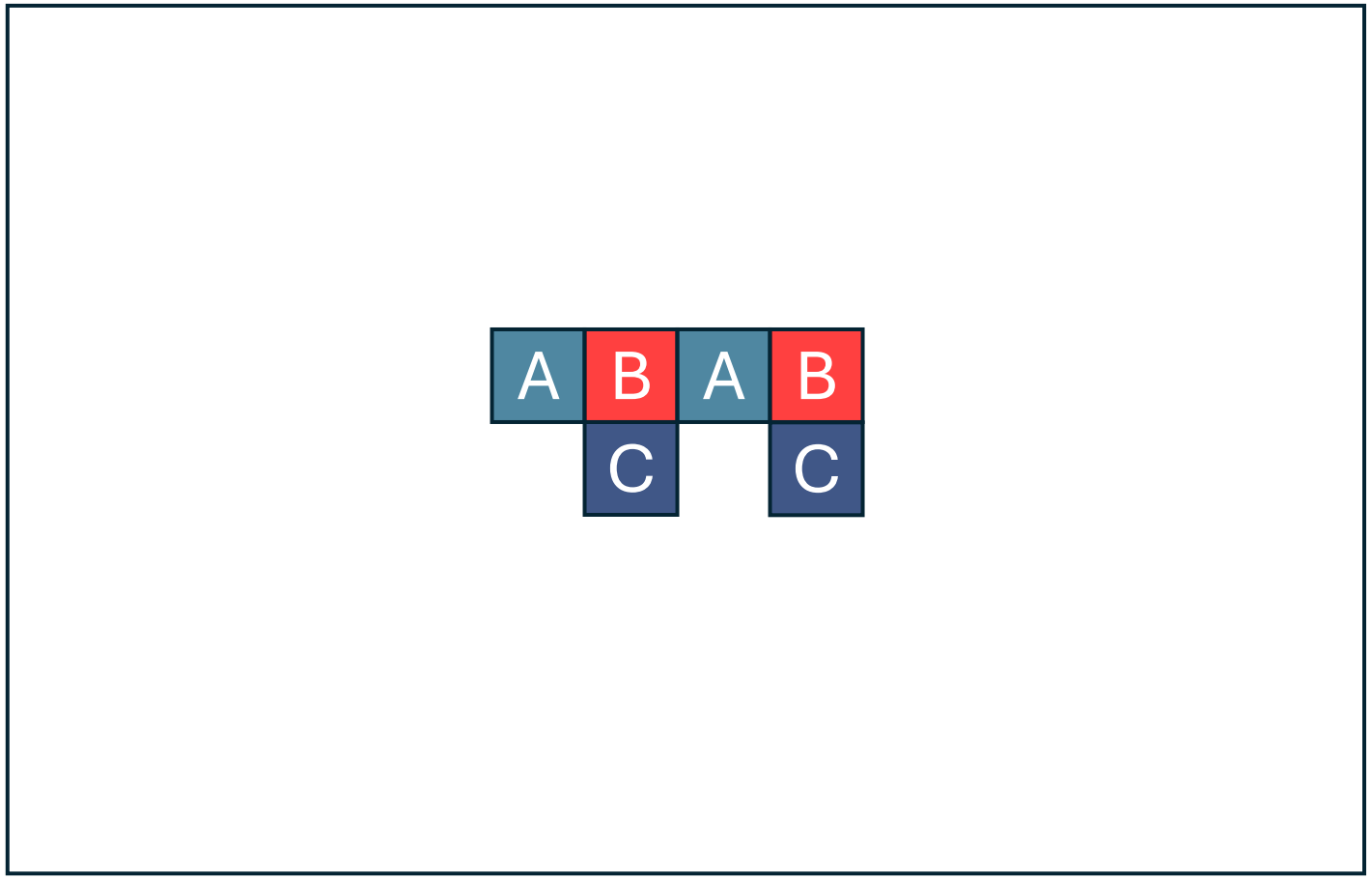
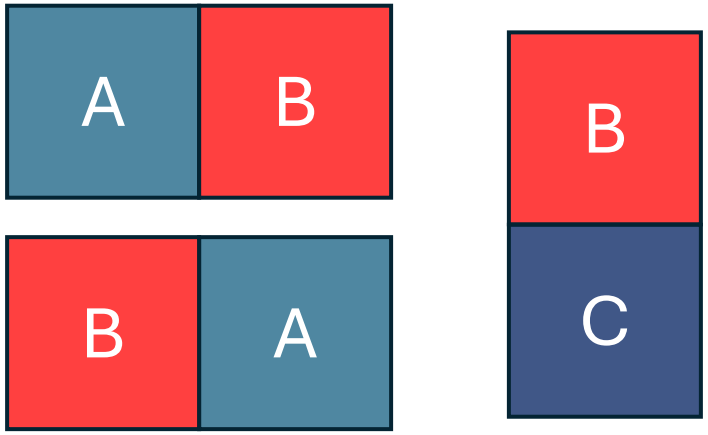


The aTAM (abstract Tile Assembly Model)

Tiles:

Affinities:

Temperature: τ



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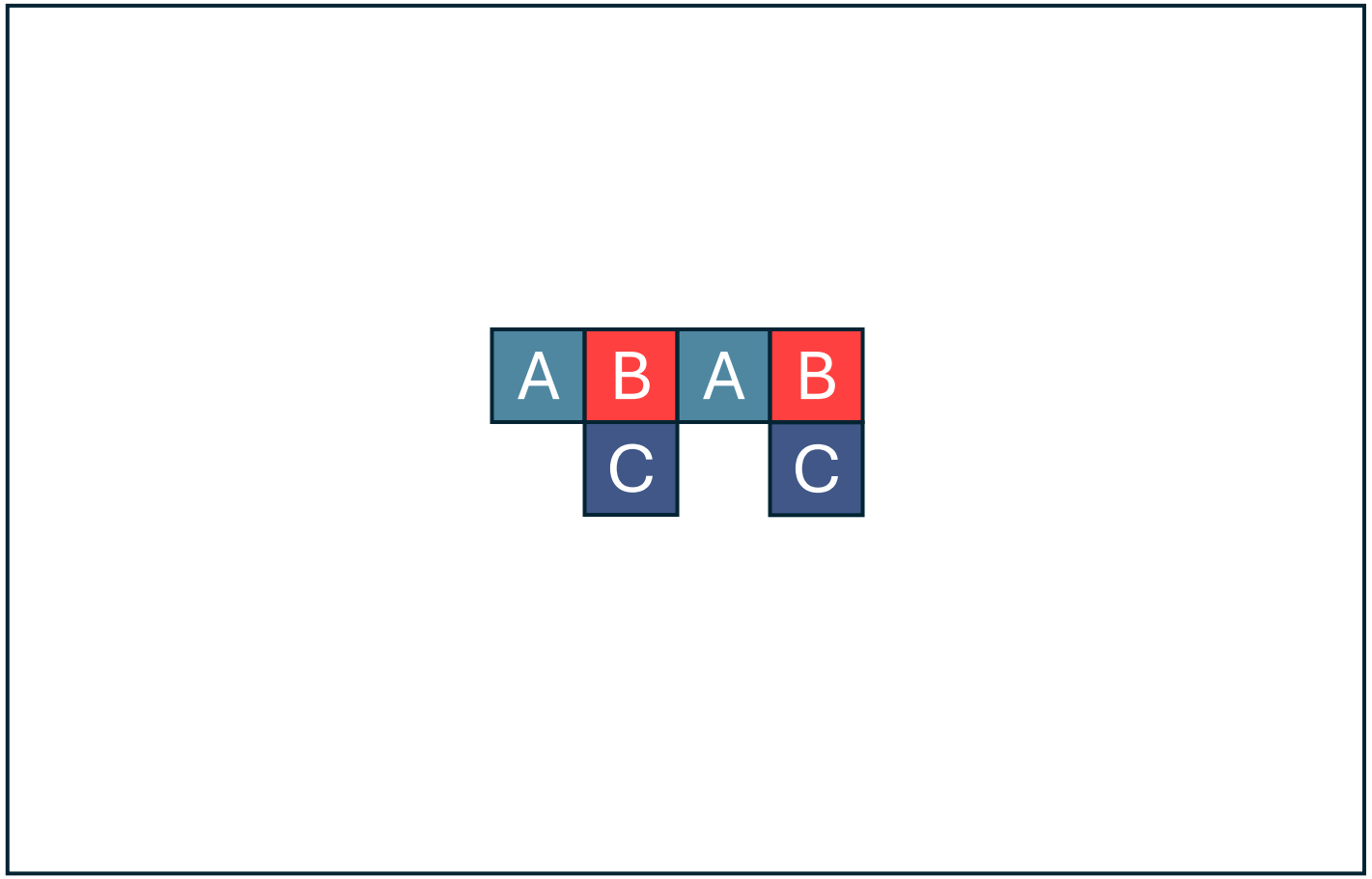

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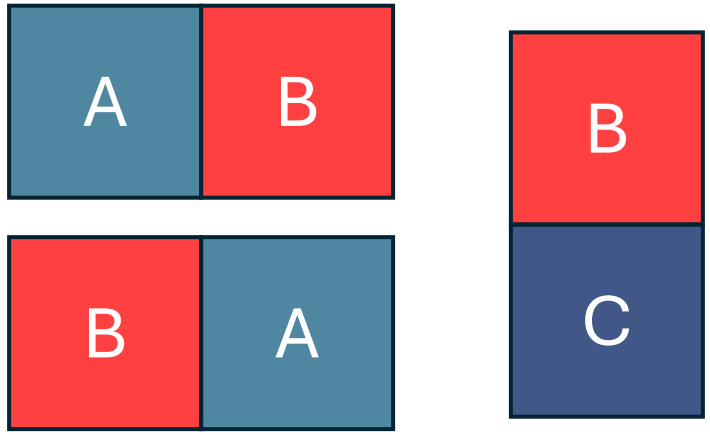

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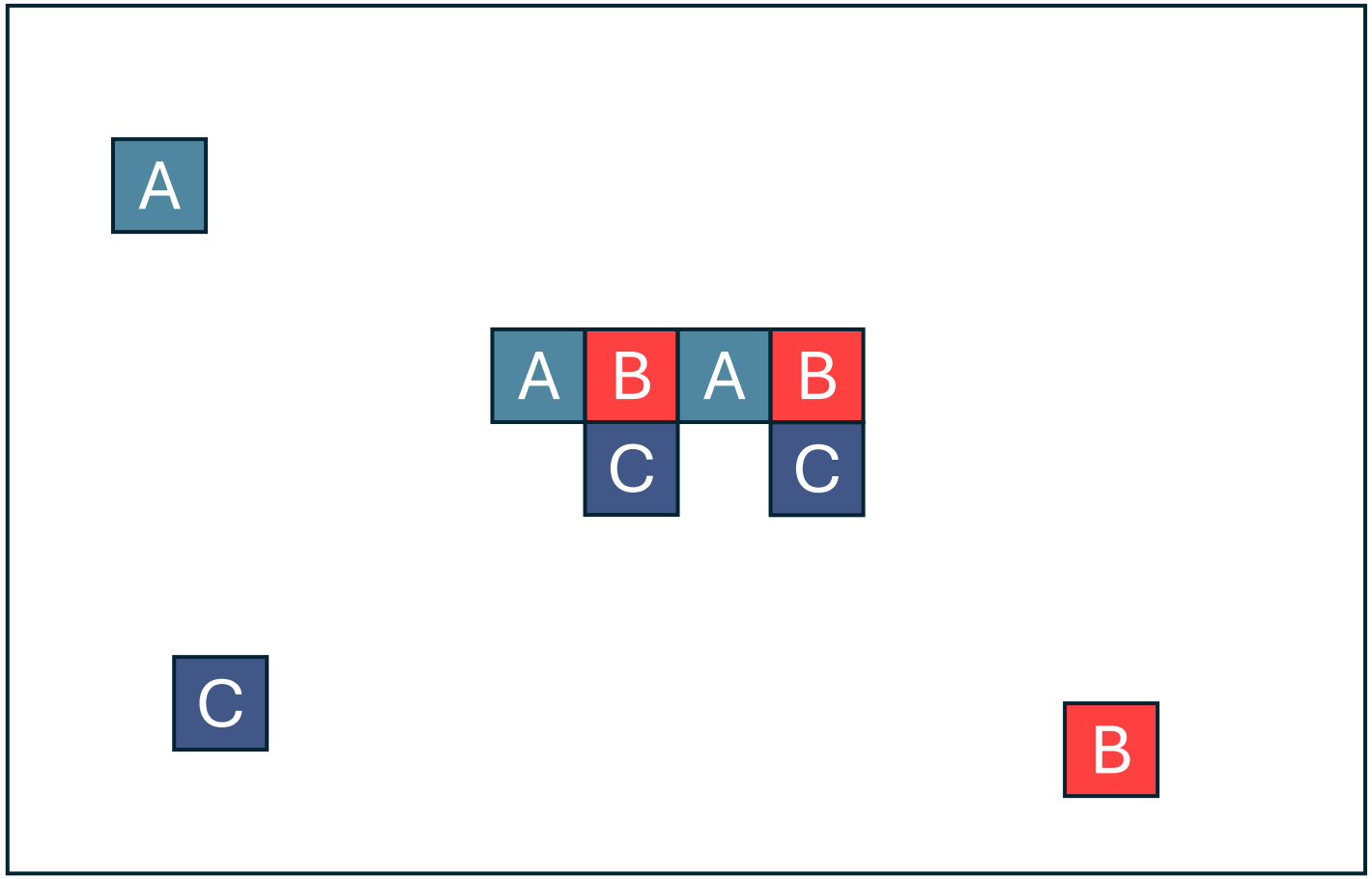
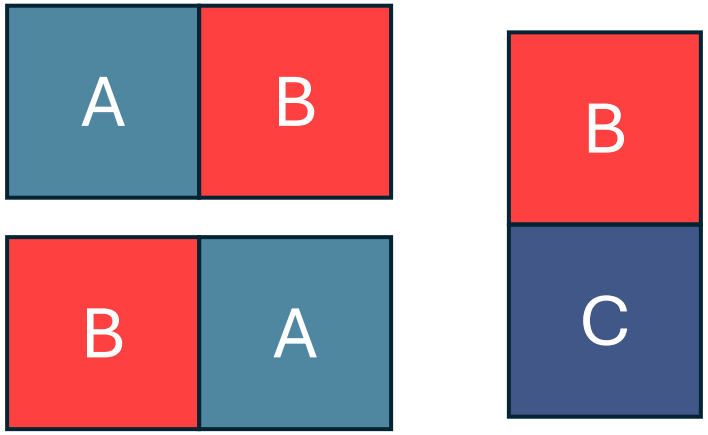


The aTAM (abstract Tile Assembly Model)

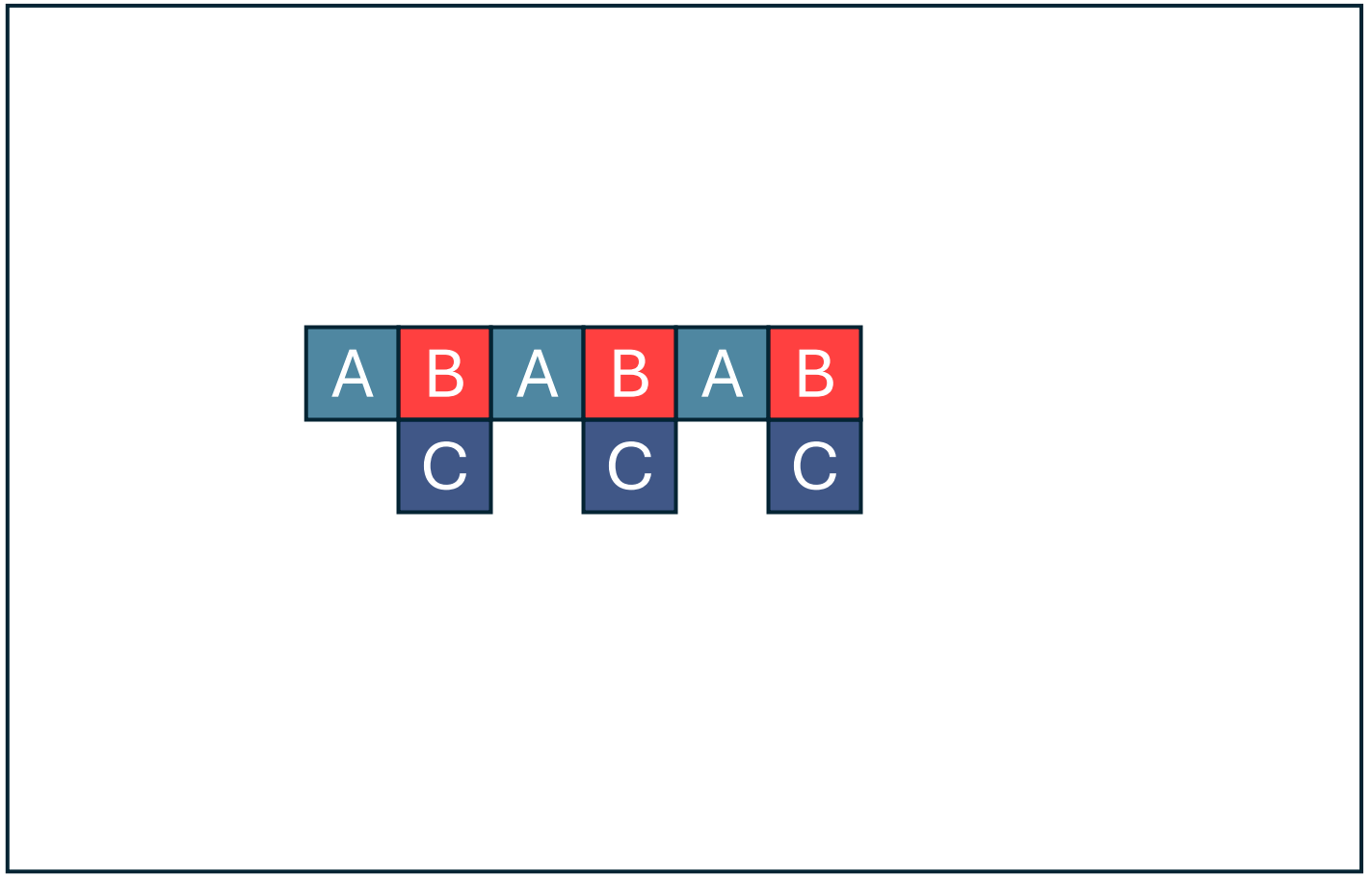
Tiles:

Affinities:

Temperature: τ



System:

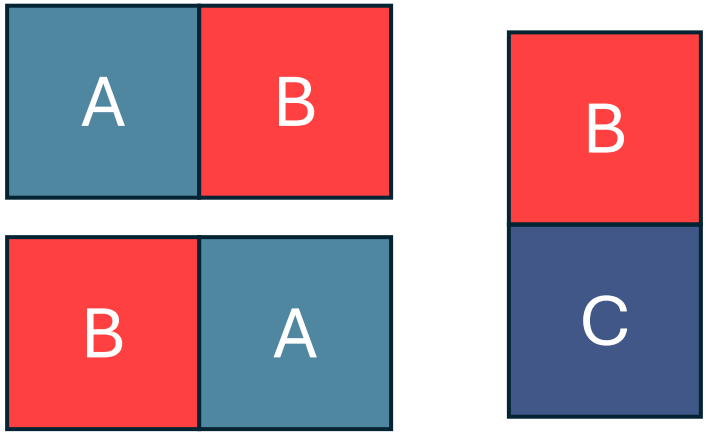


The aTAM (abstract Tile Assembly Model)

Tiles:

Affinities:

Temperature: τ



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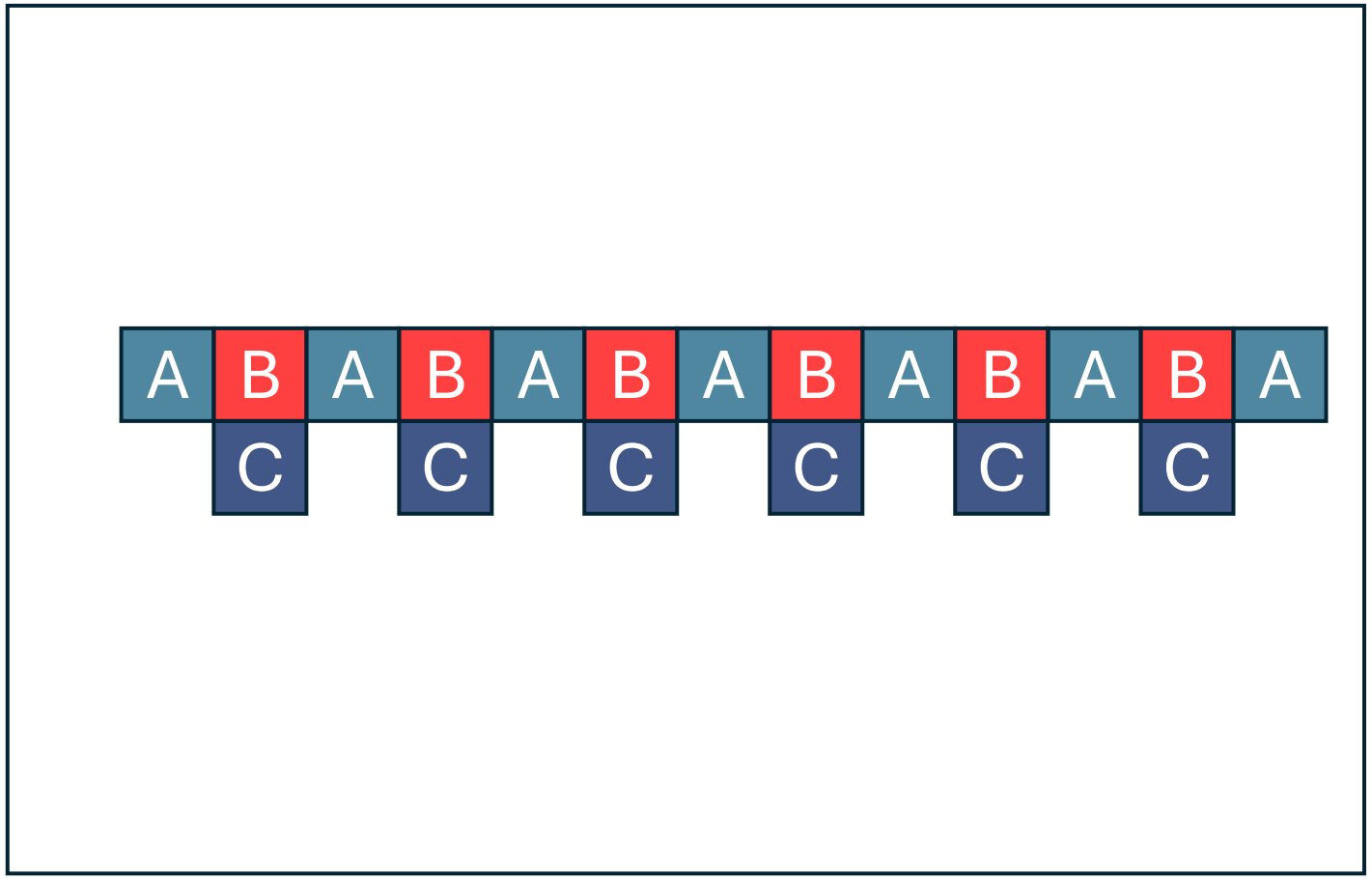
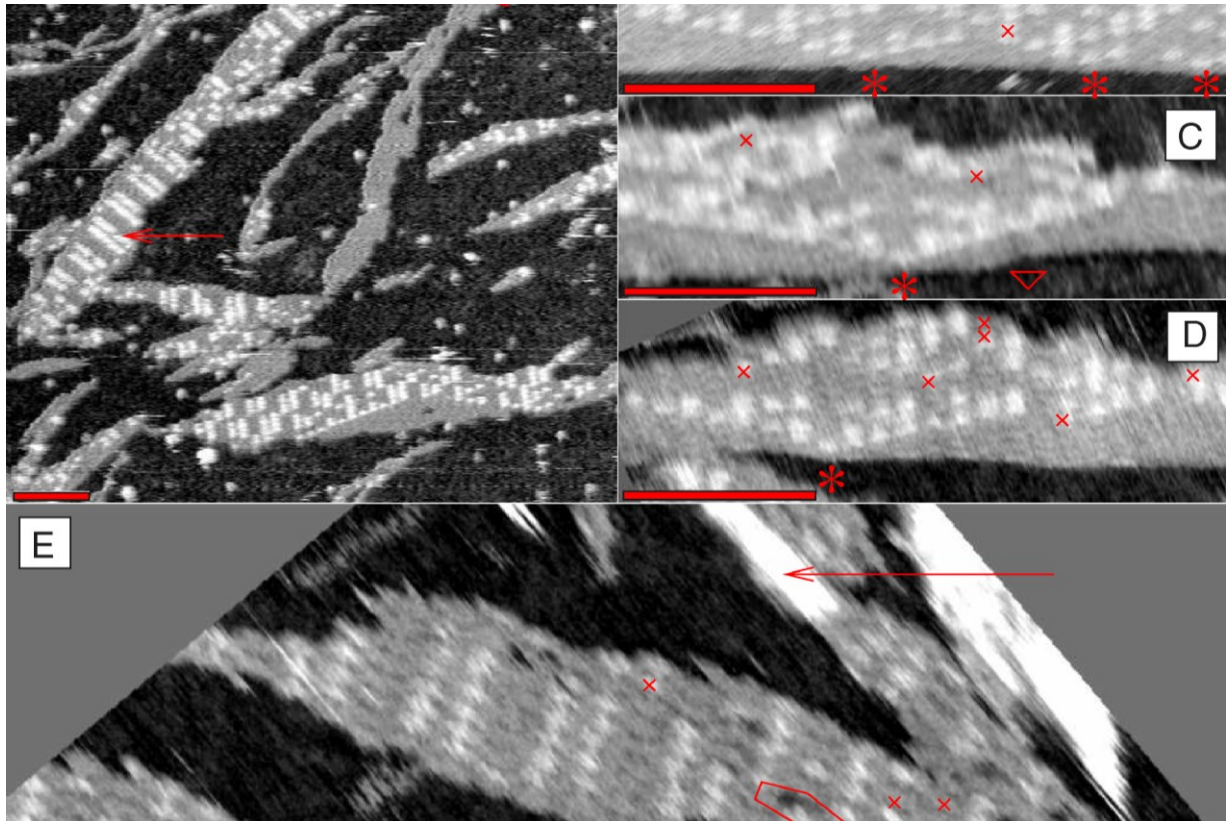
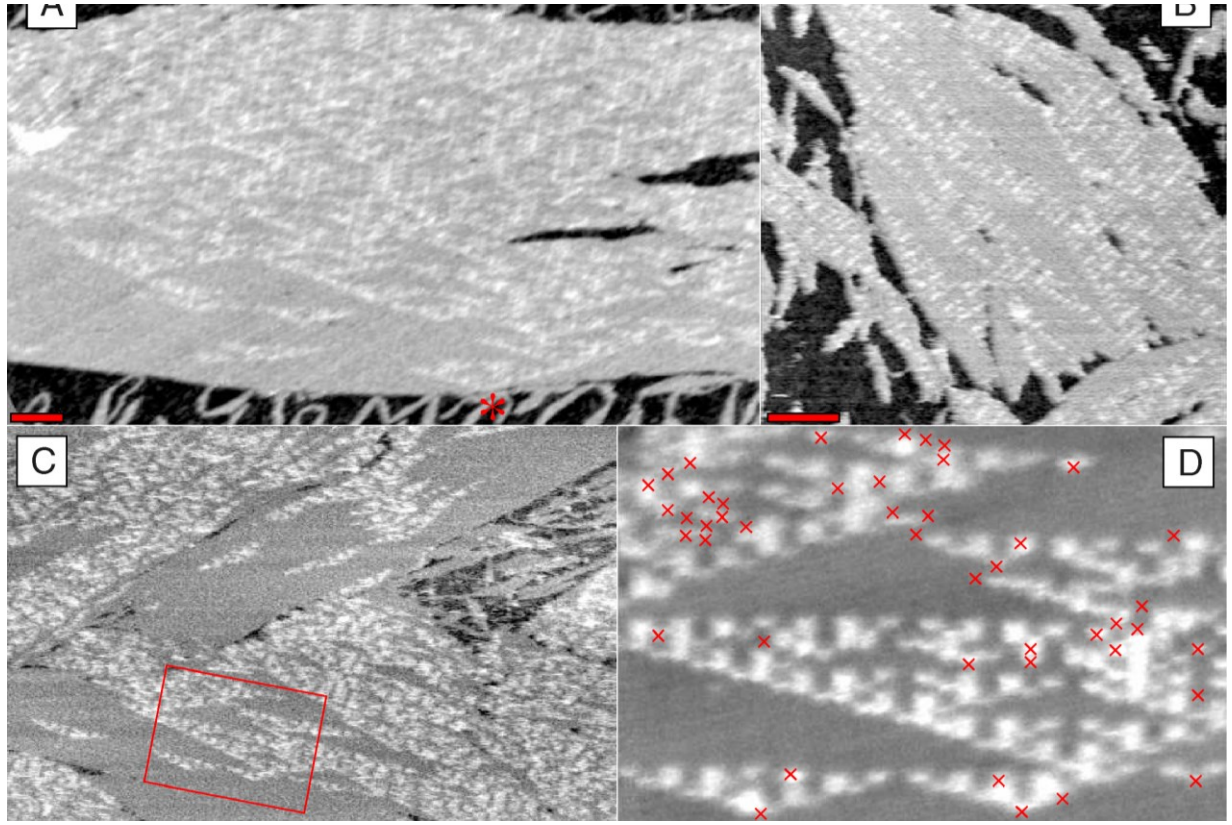


What is known (aTAM)

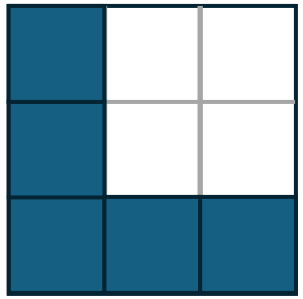


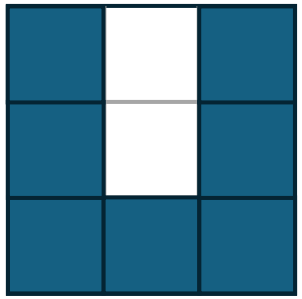


Rothemund PWK, Papadakis N, Winfree E (2004) Algorithmic Self-Assembly of DNA Sierpinski Triangles. PLoS Biol 2(12): e424.

What is known (aTAM)

- 2010:
  1. Any “L” shape does not strictly self assemble in the aTAM (any temp)
  2. No fractal weakly self assembles at temperature 1
  3. “Nice” fractals can be weakly self assembled at higher temps





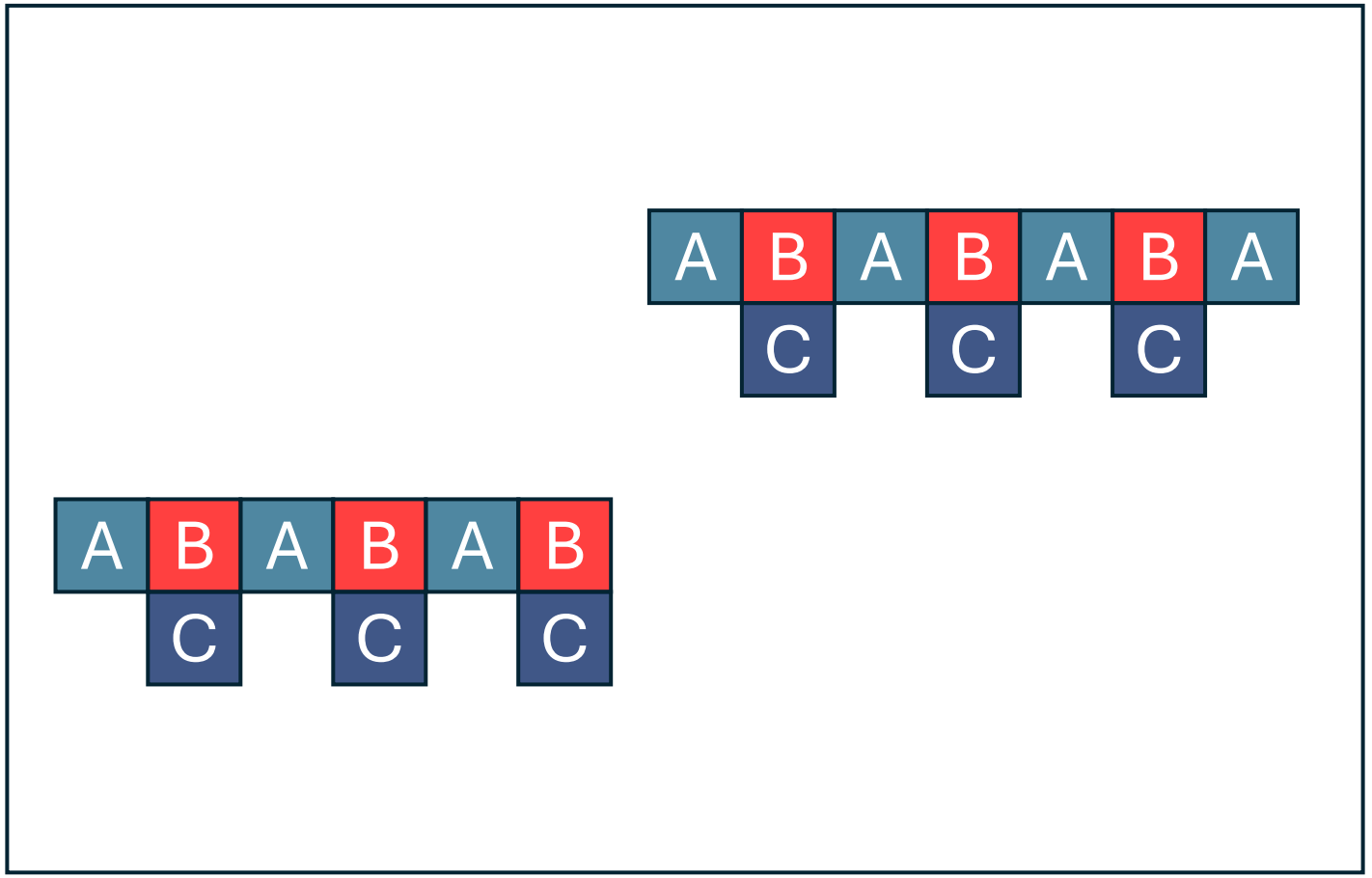
- 2020:
  1. The “H” and “U” shapes do not strictly self assemble in the aTAM

Variations of the aTAM

- 2HAM (2-Handed Assembly Model):
    1. 2 different assemblies can attach

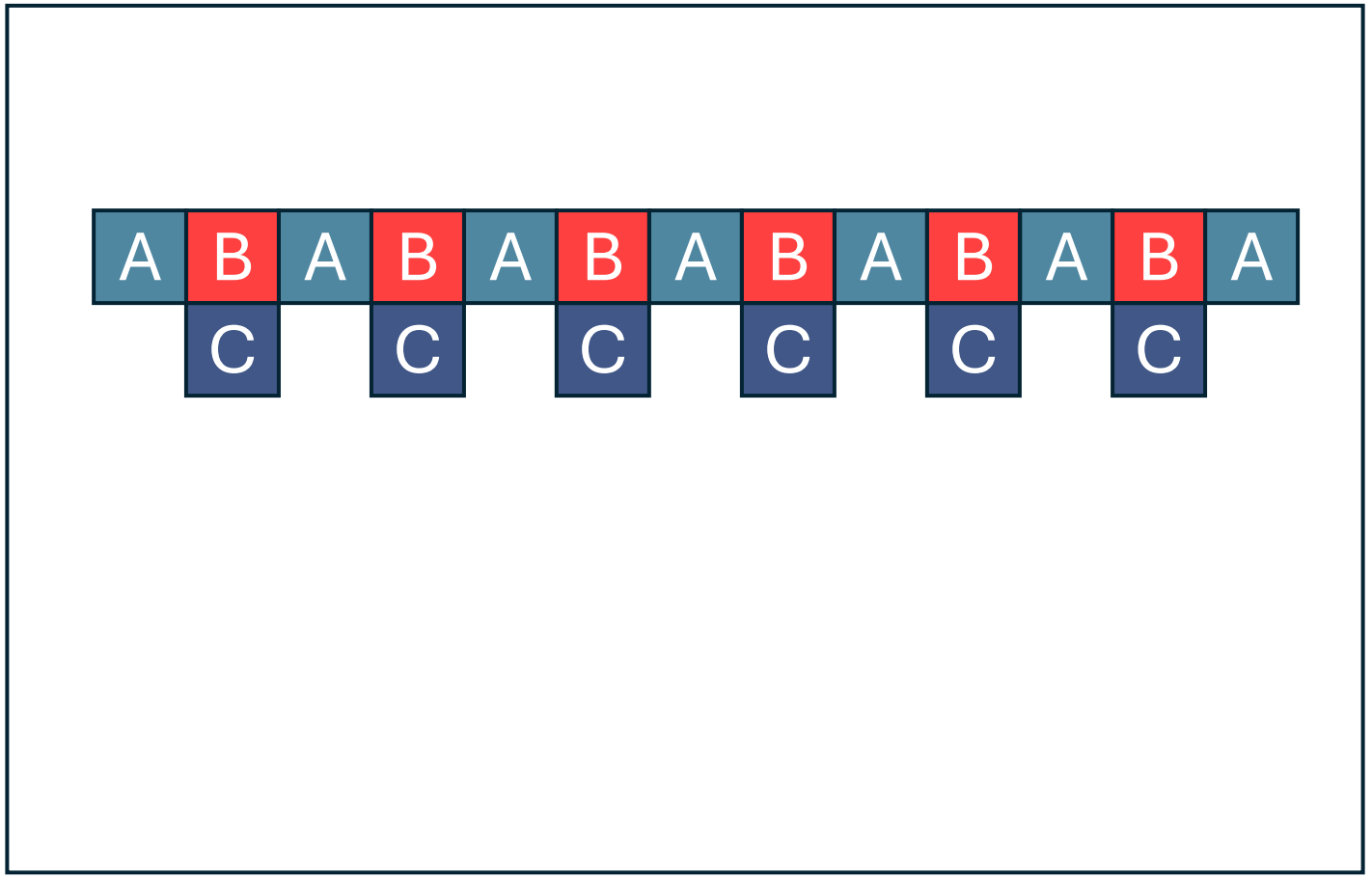
The 2HAM (2-Handed Assembly Model)

System:



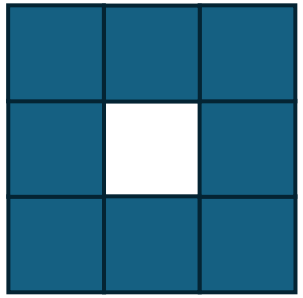
The 2HAM (2-Handed Assembly Model)

System:



Variations of the aTAM

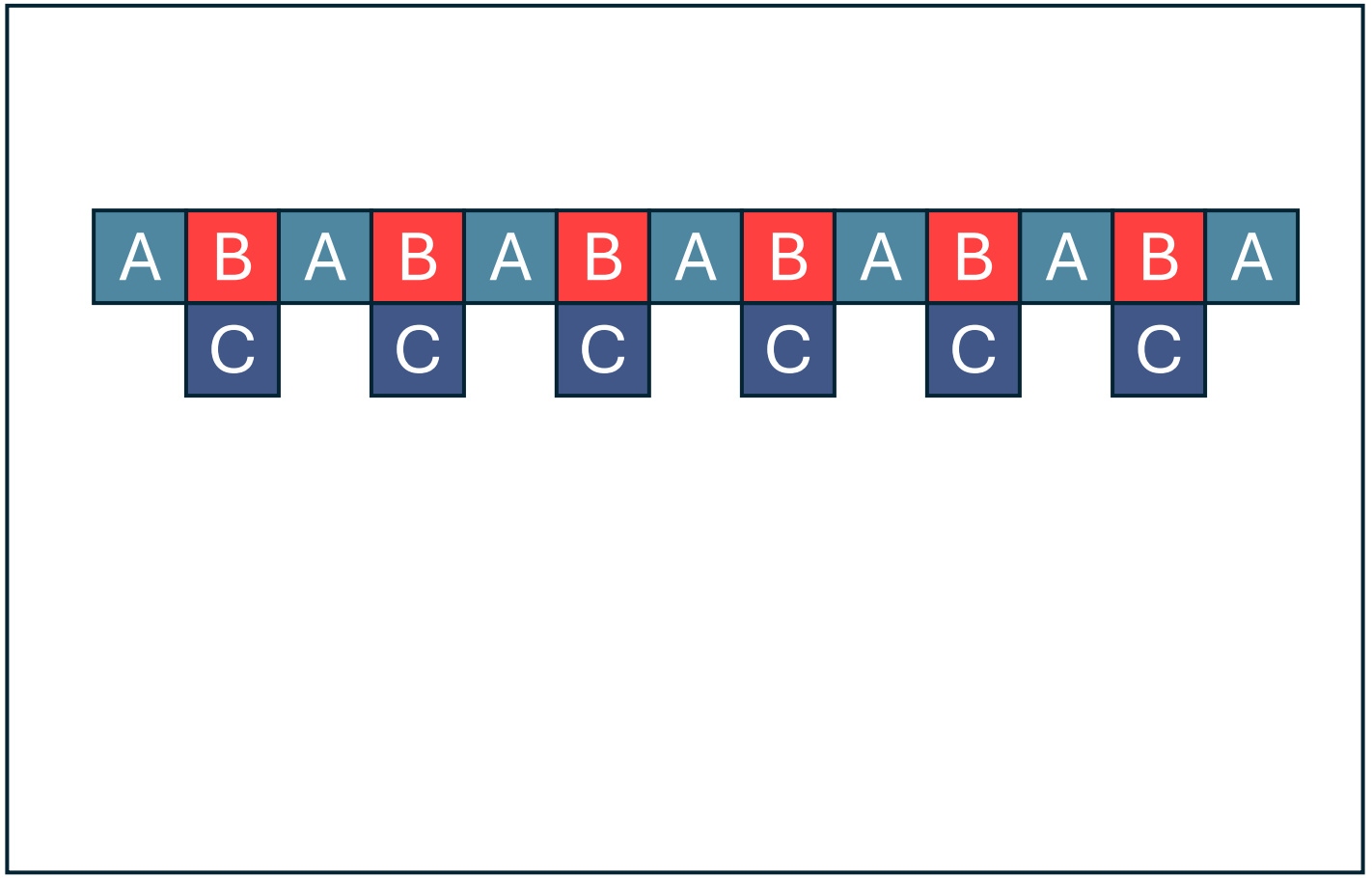
- 2HAM (2-Handed Assembly Model):
  1. 2 different assemblies can attach
  2. Can finitely assemble a larger class of shapes



- STAM (Signal-Passing Tile Assembly Model):
  1. Glues can turn “on” and “off”
  2. Detachment is allowed

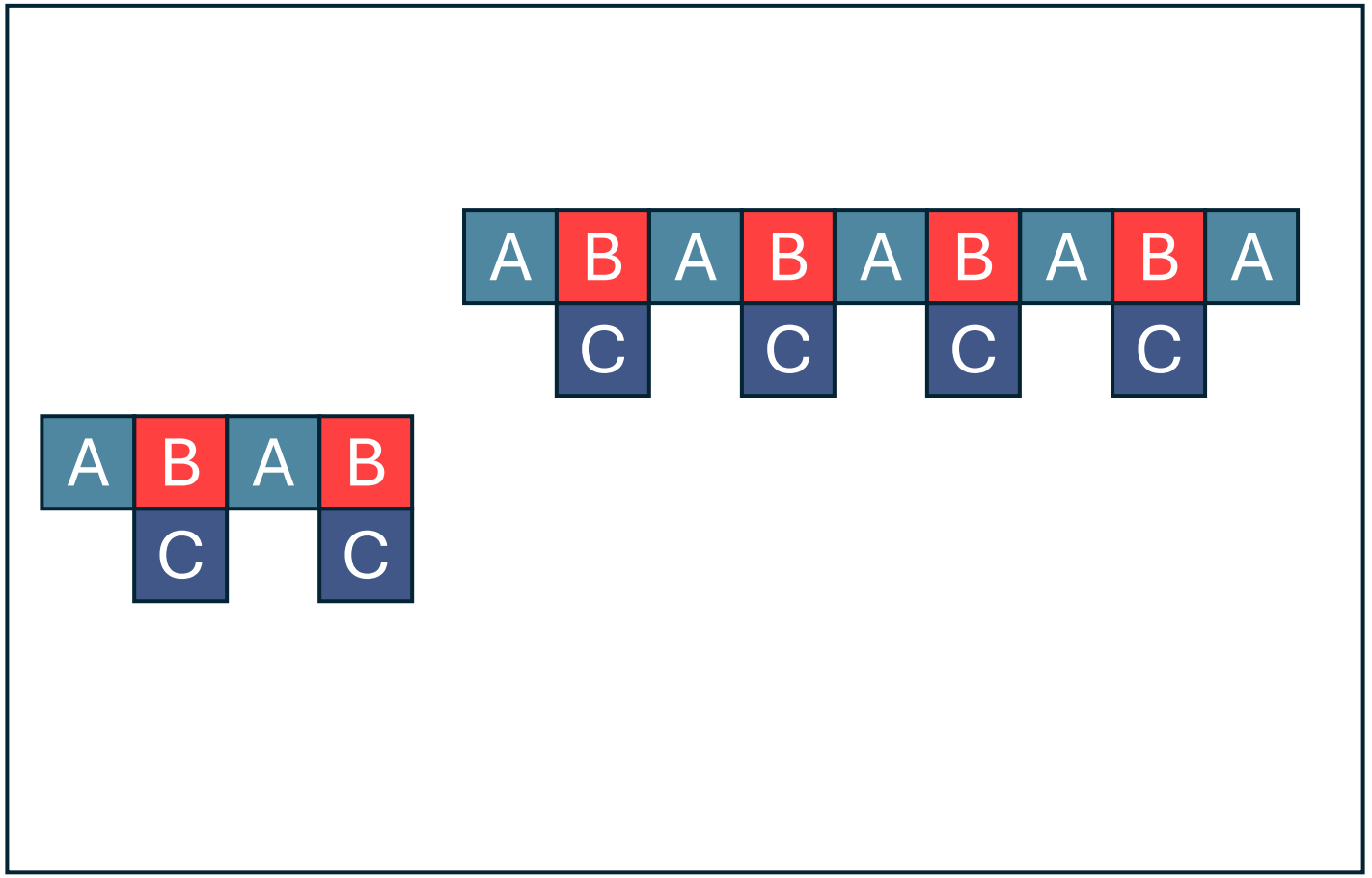
The STAM (Signal-Passing Tile Assembly Model)

System:



The STAM (Signal-Passing Tile Assembly Model)

System:



Variations of the aTAM

- 2HAM (2-Handed Assembly Model):
  1. 2 different assemblies can attach
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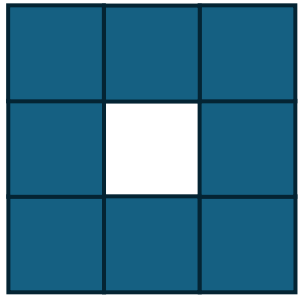


- STAM (Signal-Passing Tile Assembly Model):
  1. Glues can turn “on” and “off”
  2. Detachment is allowed
  3. Any arbitrary fractal is buildable
  4. Without detachments, some shapes impossible

A Comparison

	aTAM	2HAM	STAM
Description	- Single tile attachments - No detachments - Fixed glues	- Up to 2 assembly attachments - No detachments - Fixed glues	- Up to 2 assembly attachments - Detachments - Glues turn on/off
Results	- No fractals weakly buildable (temp 1) - Some fractals not strictly buildable (temp > 1)	- Finitely assemble larger class of shapes - Still some non-buildable fractals	- All fractals strictly buildable - **Without detachment:** some impossible fractals

Question: What if adjacent tiles can infinitely change states?

Seeded Tile Automata

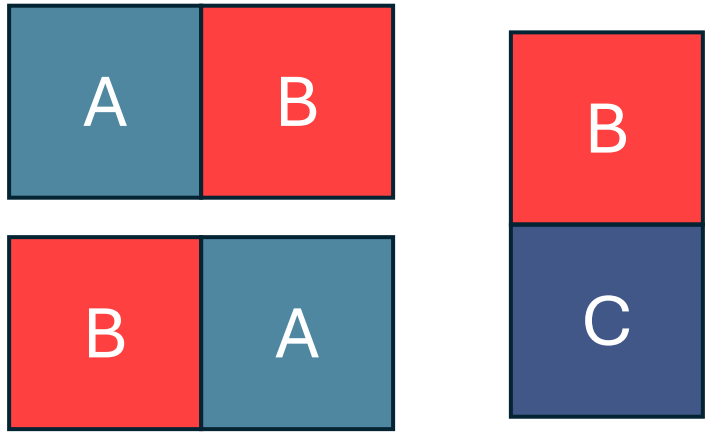
What is the model?

Seeded TA

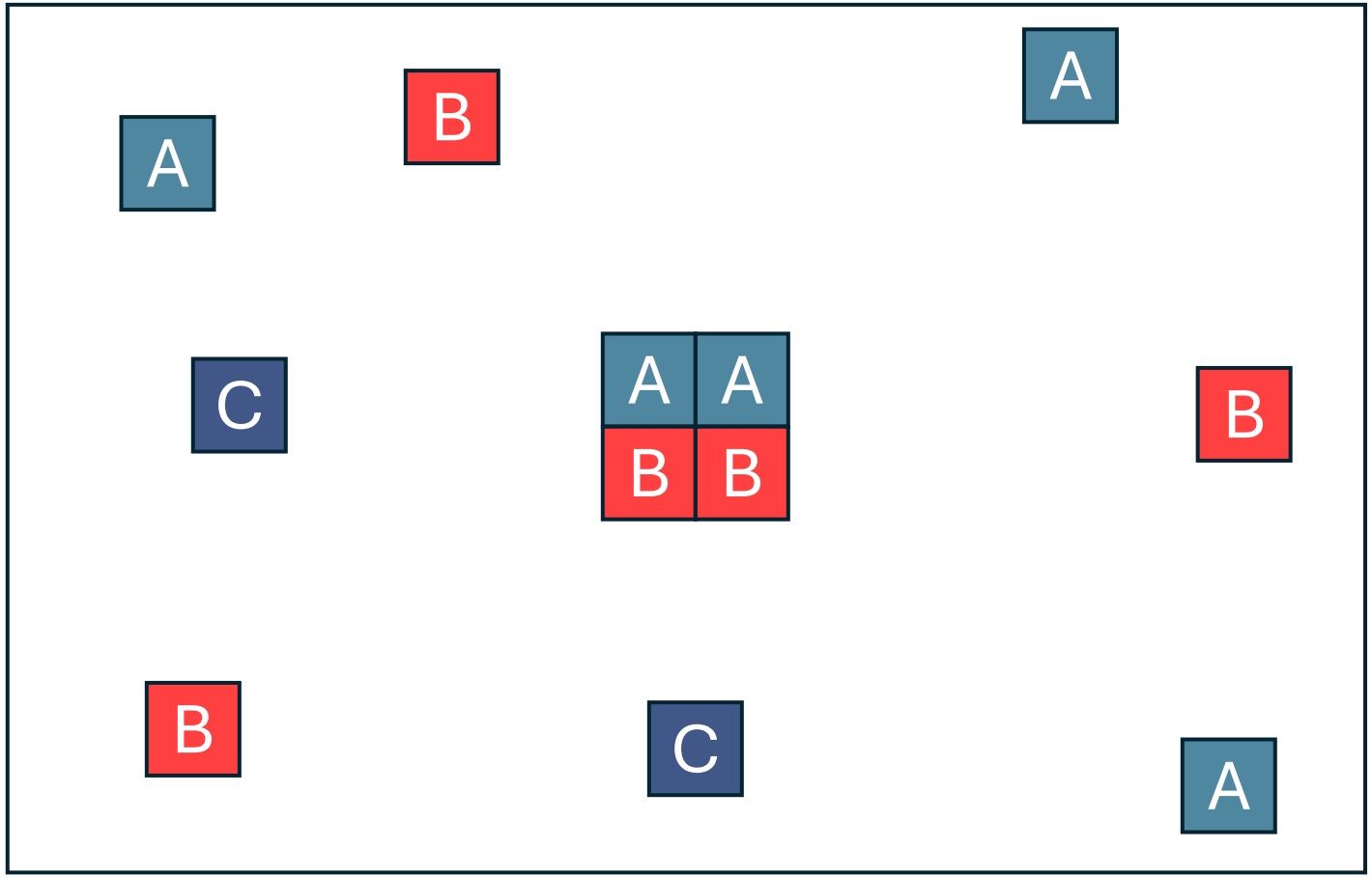
Tiles:

Affinities:

Temperature: τ



System:

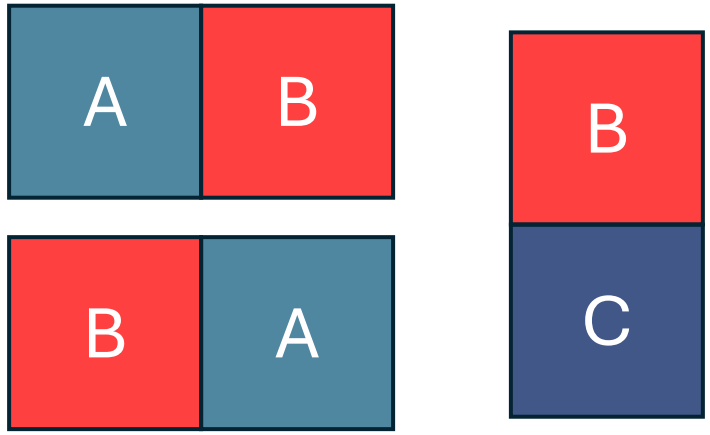


Seeded TA

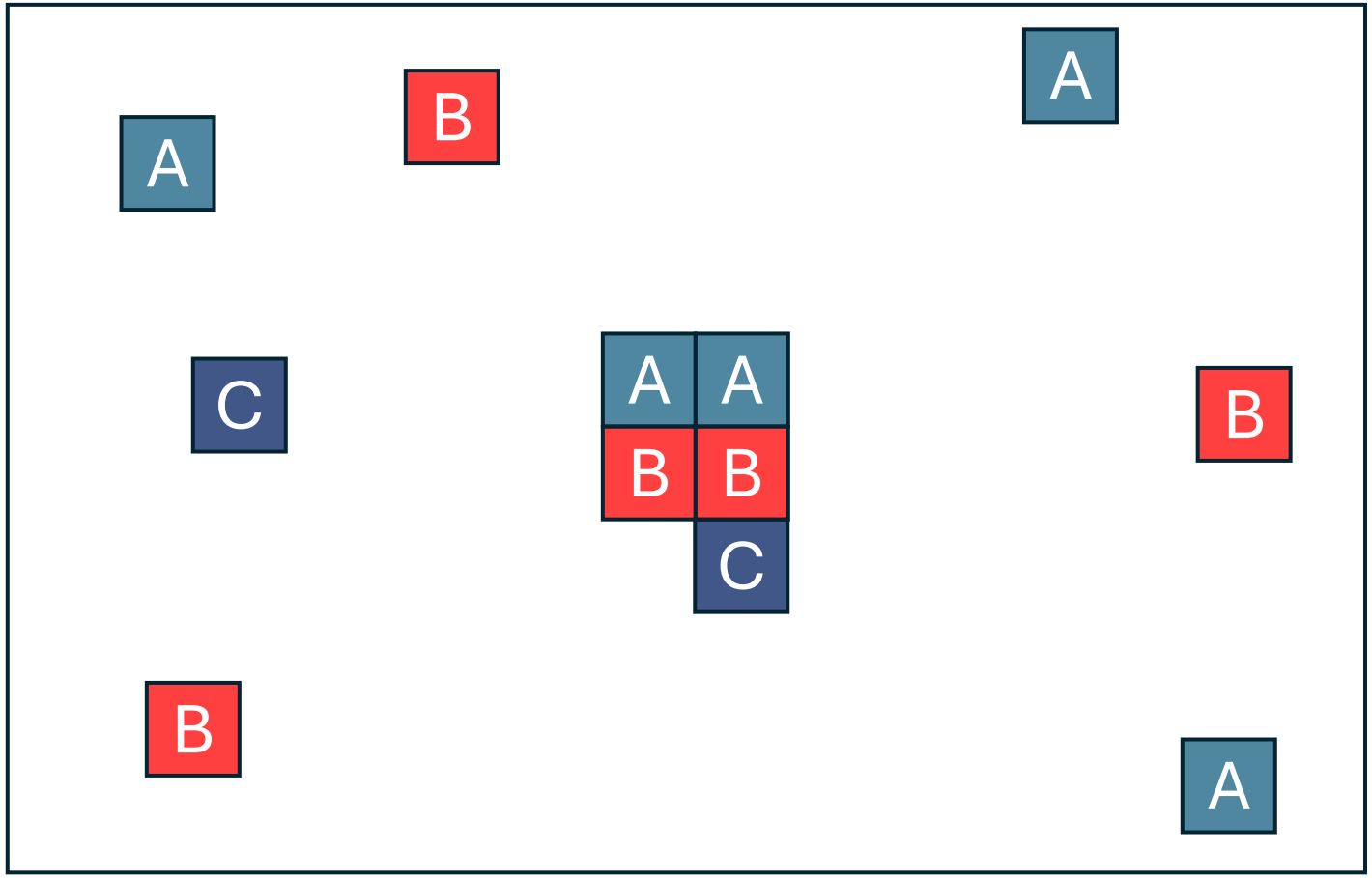
Tiles:

Affinities:

Temperature: τ



System:

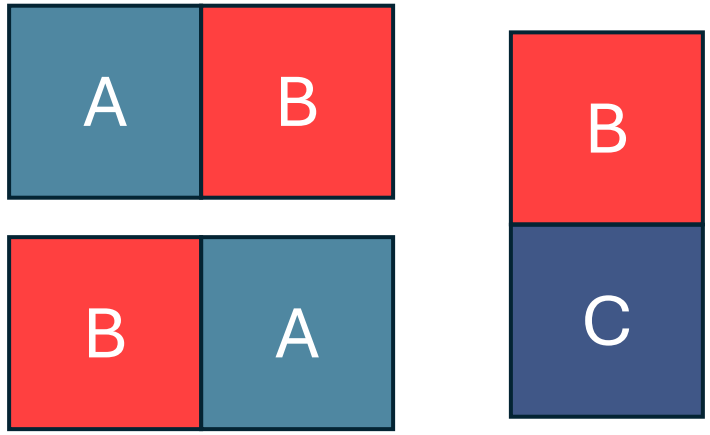


Seeded TA

Tiles:

Affinities:

Temperature: τ



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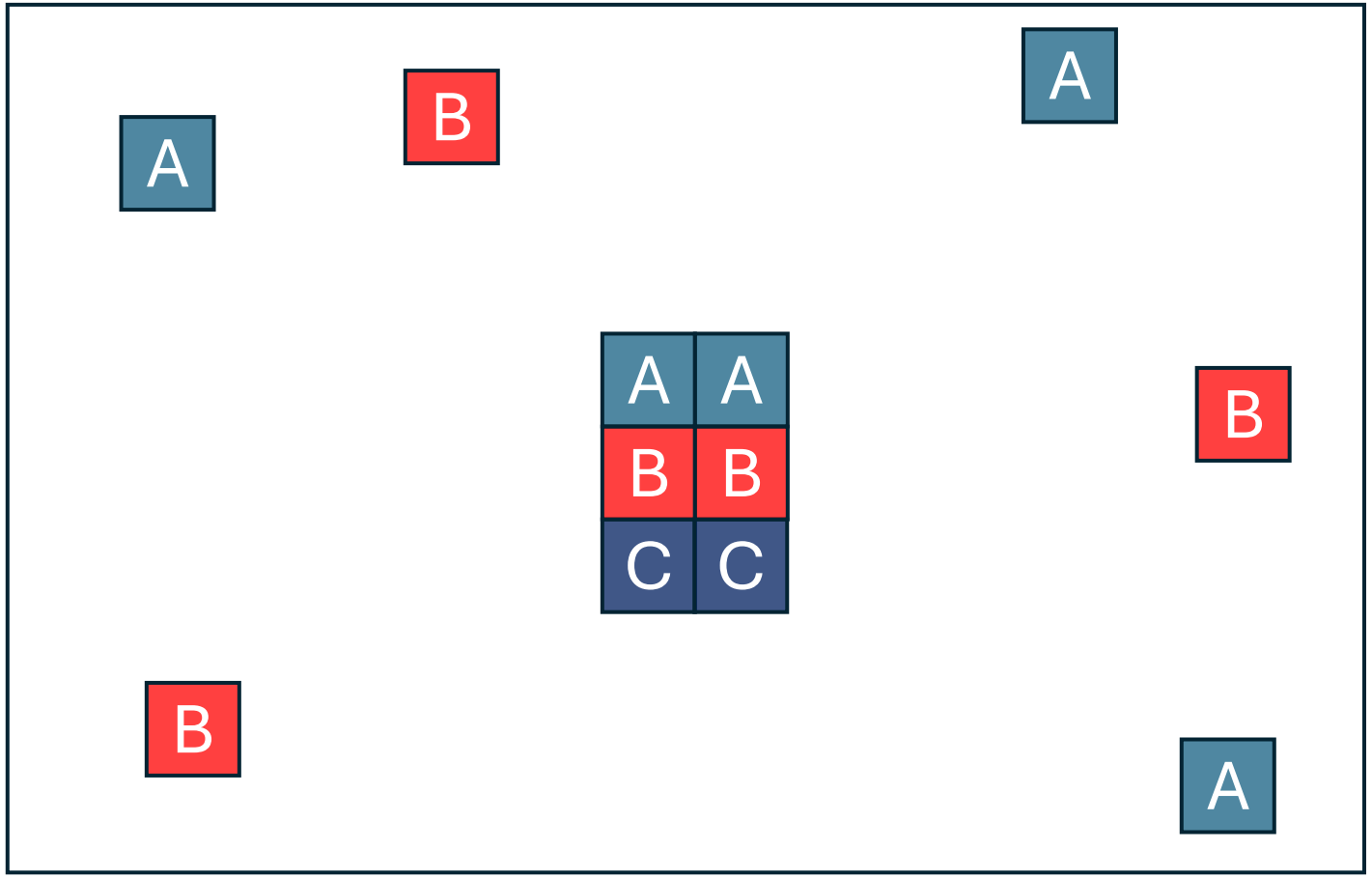
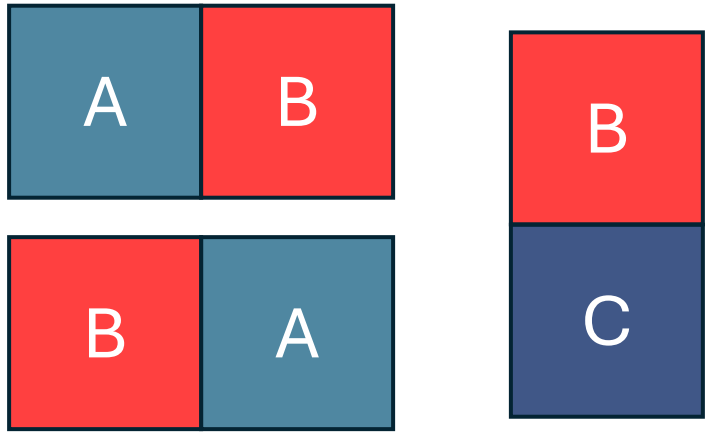


Seeded TA

Tiles:

Affinities:

Temperature: τ



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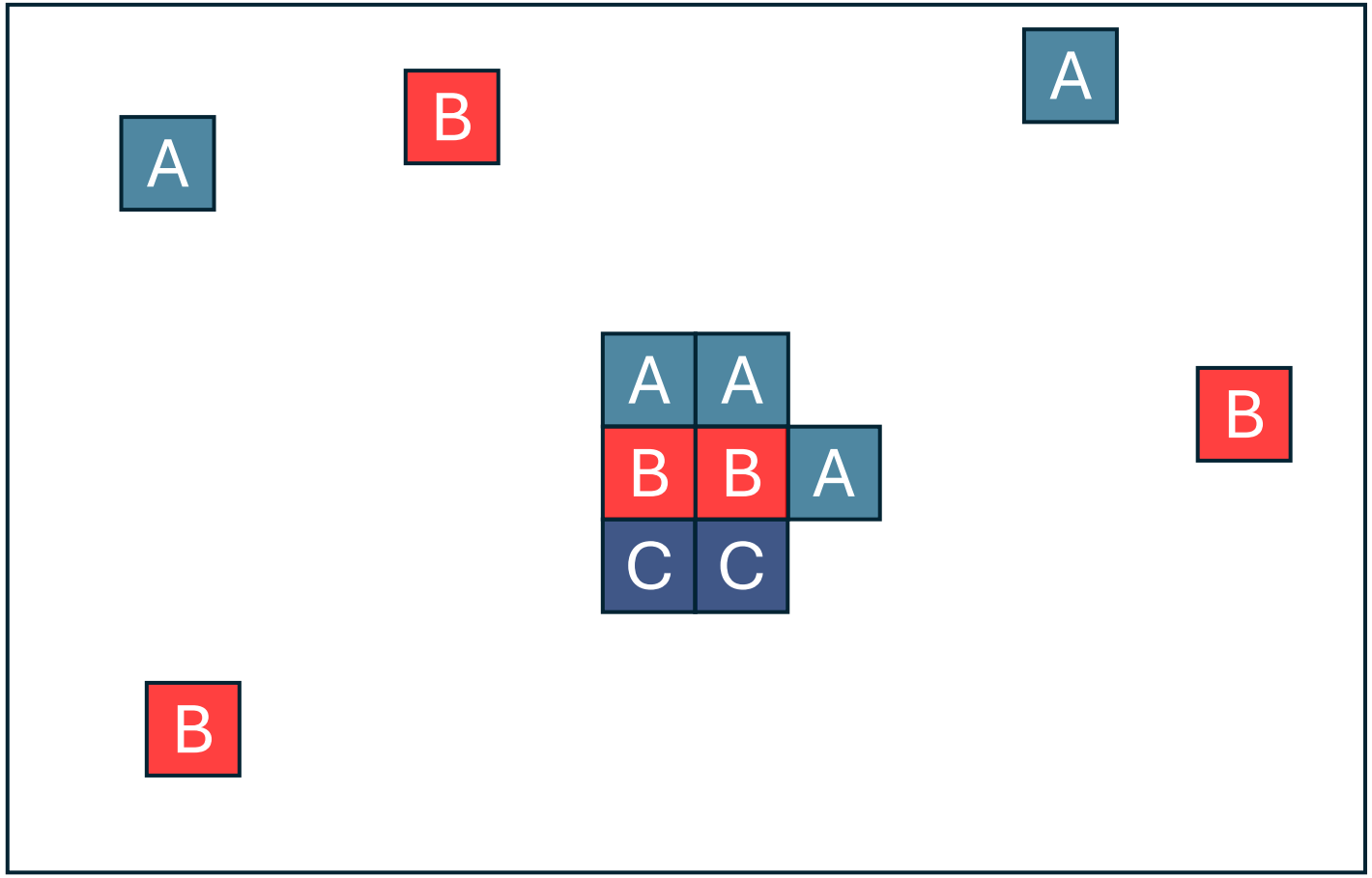
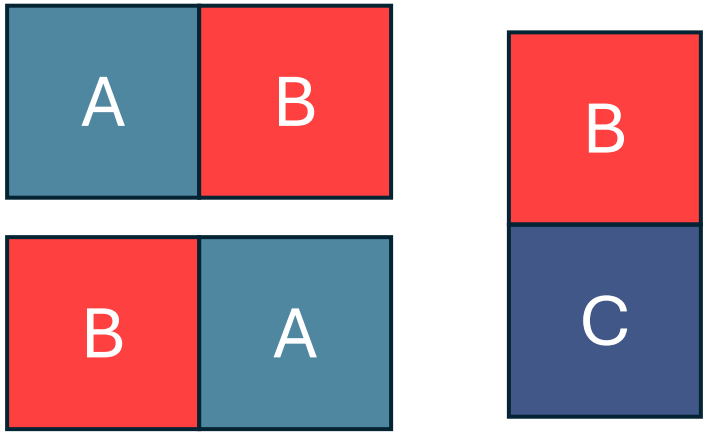


Seeded TA

Tiles:

Affinities:

Temperature: τ



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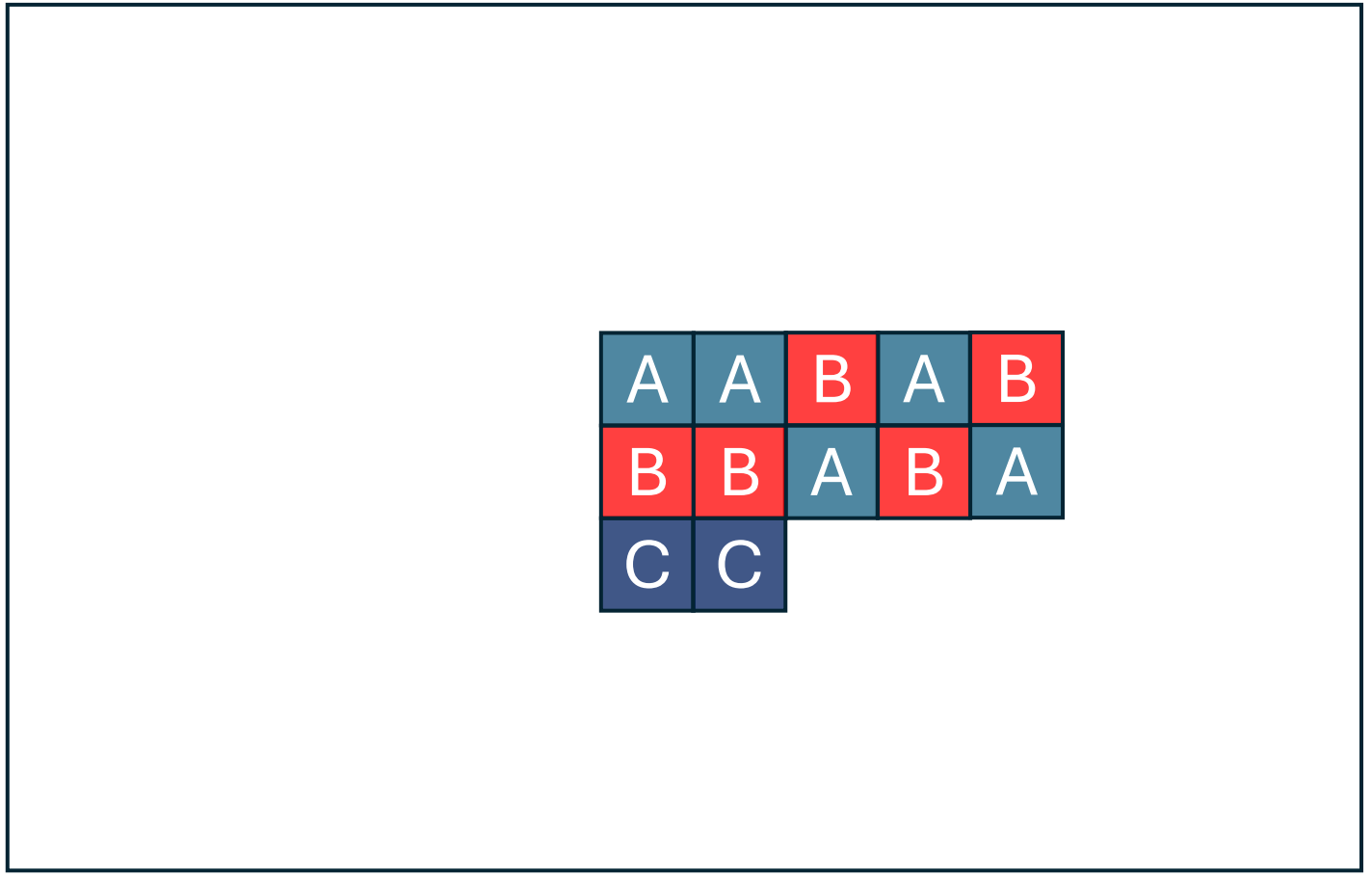
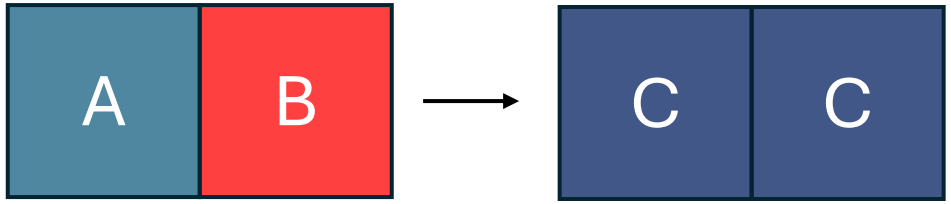


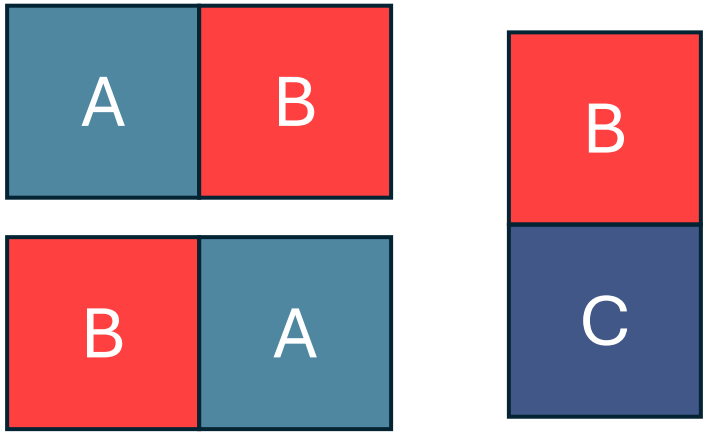
Seeded TA

Transitions:



Affinities:

Temperature: τ



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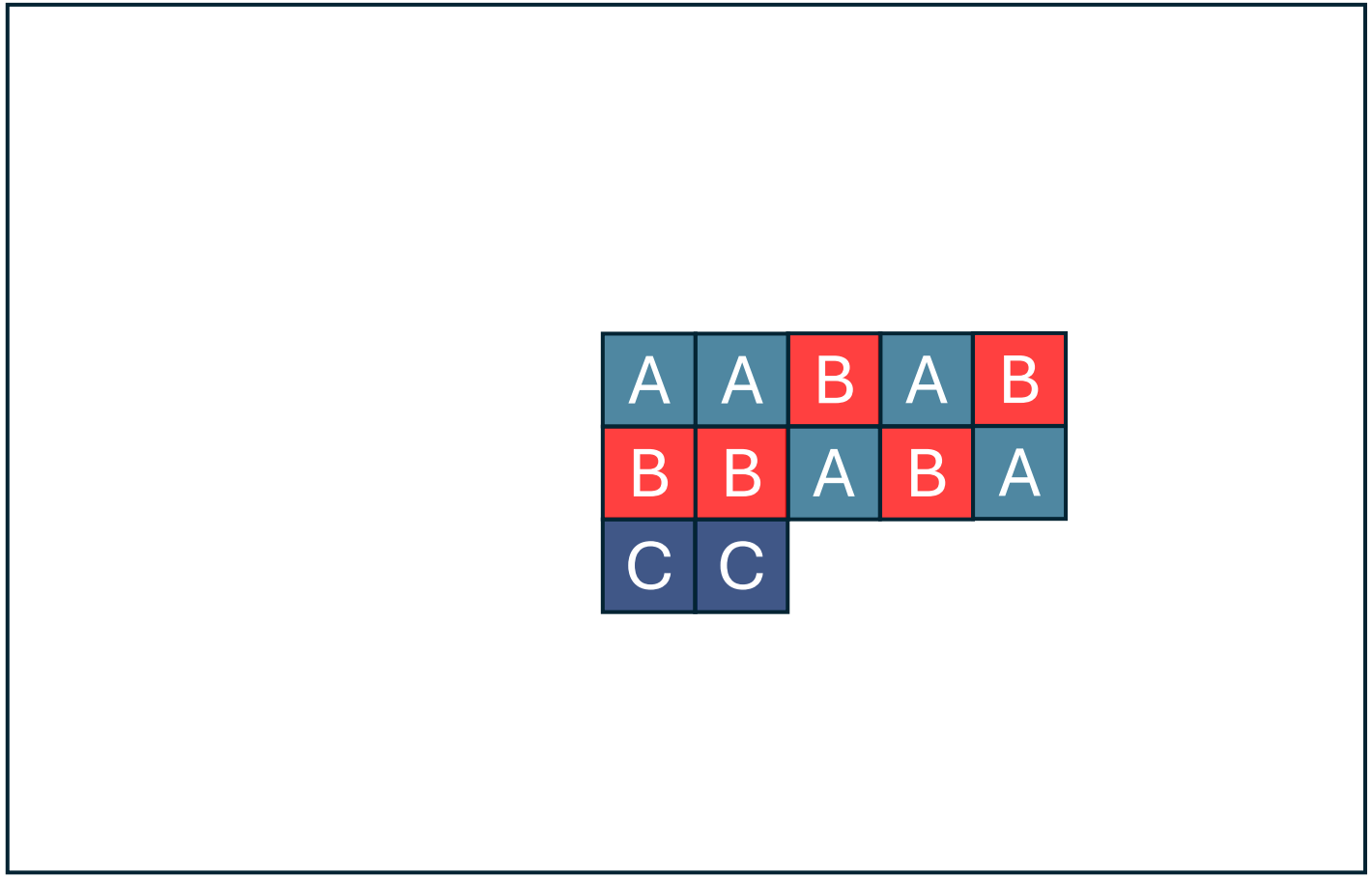
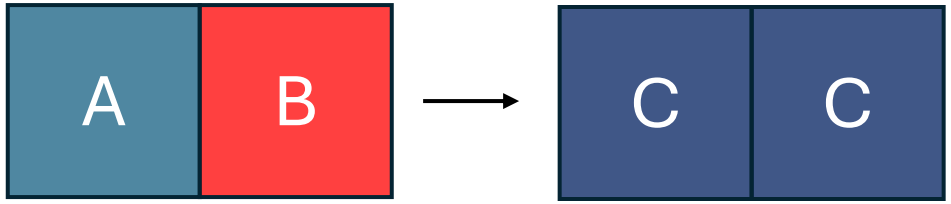


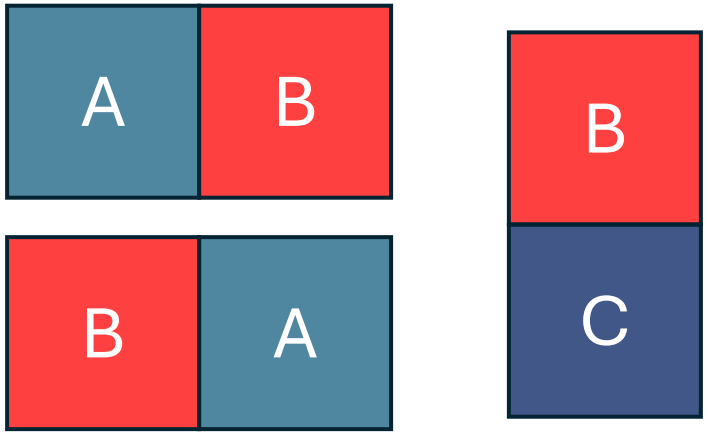
Seeded TA

Transitions:



Affinities:

Temperature: τ



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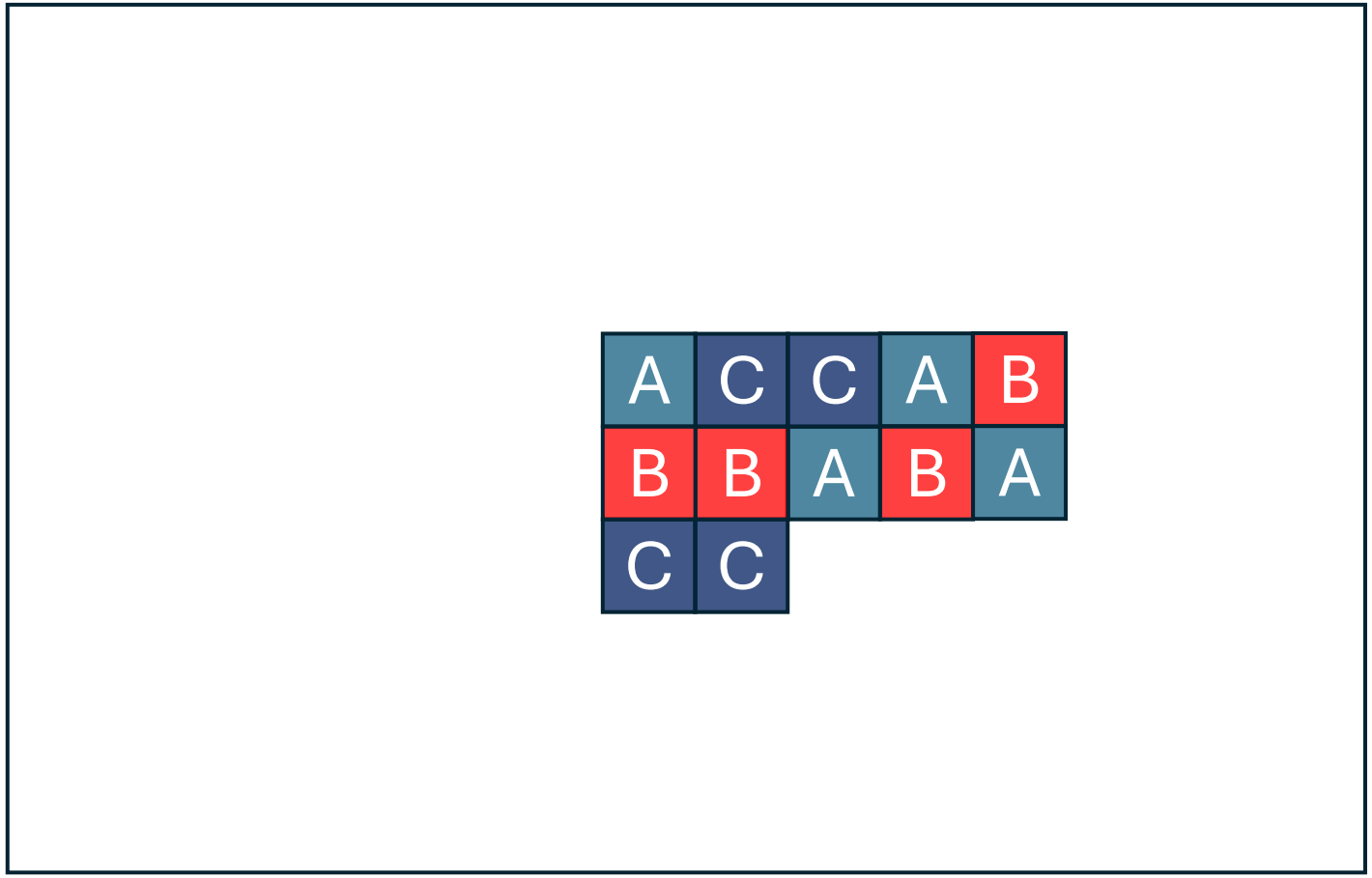

Seeded TA

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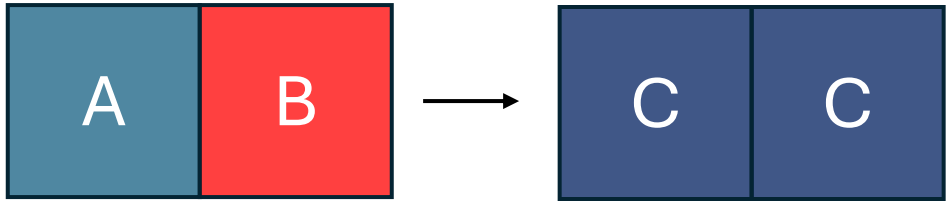

Affinities:

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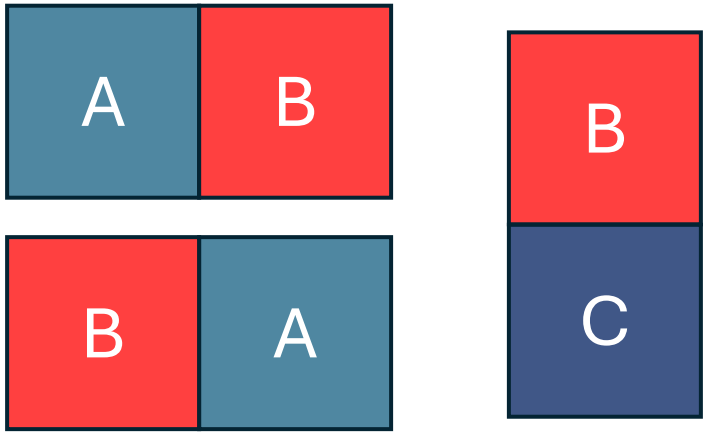

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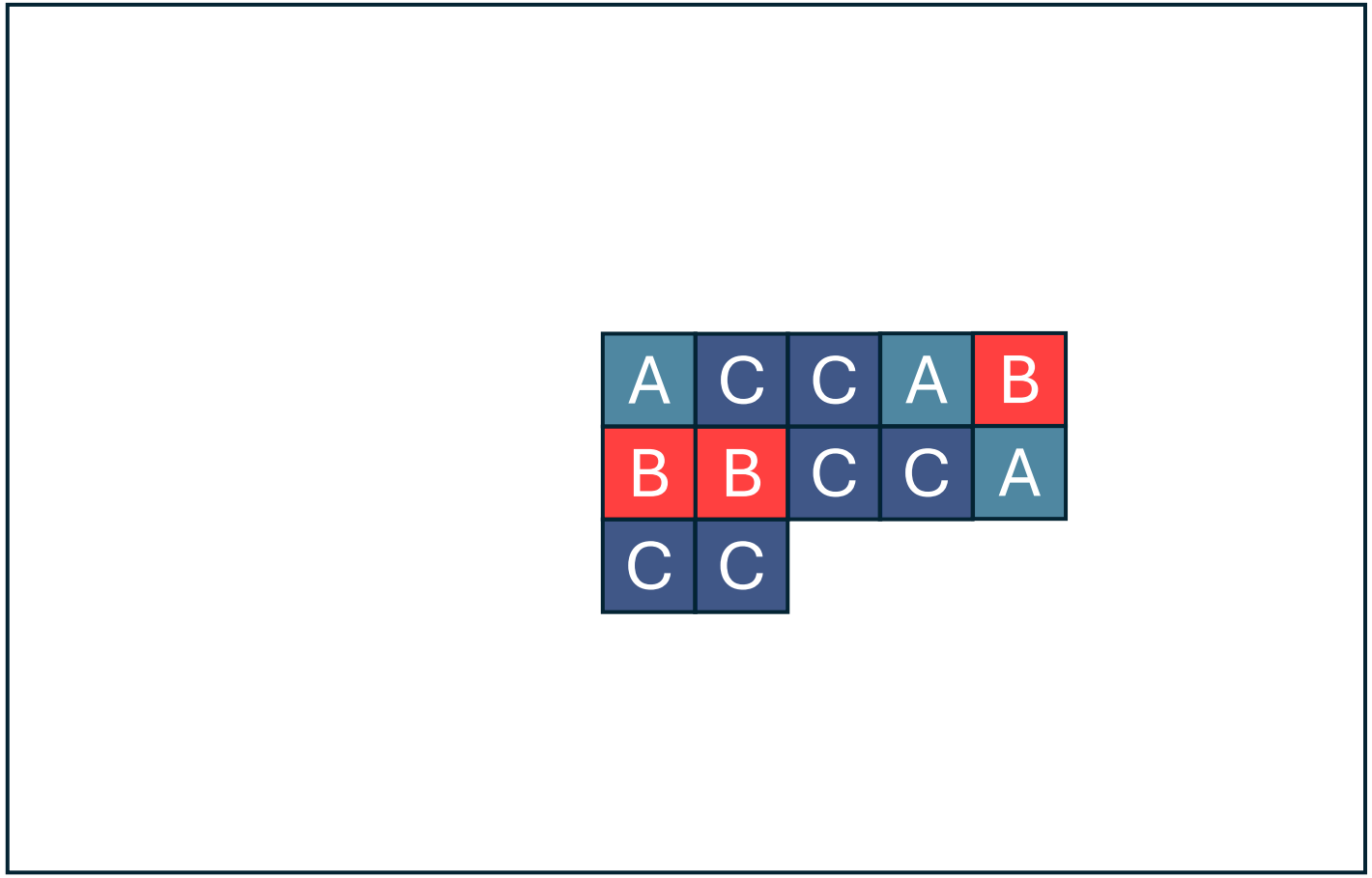
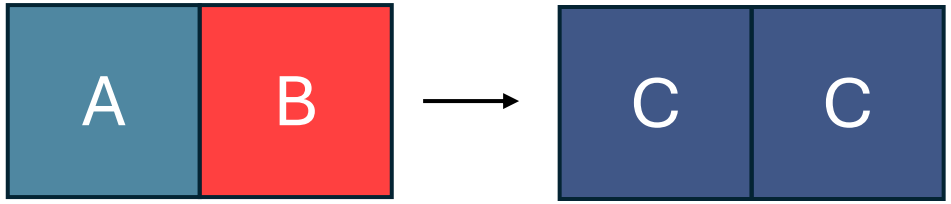


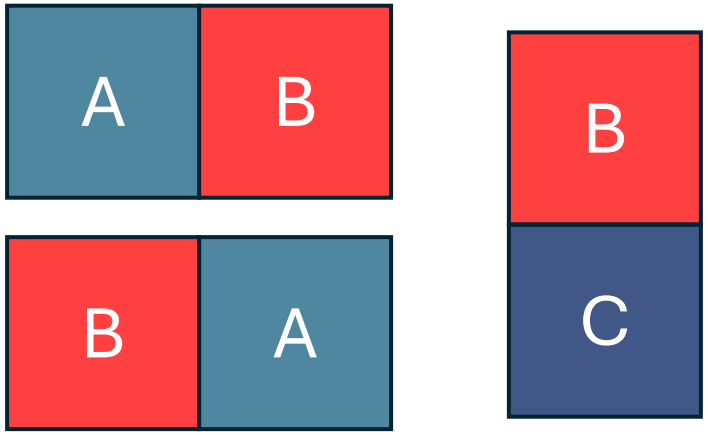
Seeded TA

Transitions:



Affinities:

Temperature: τ



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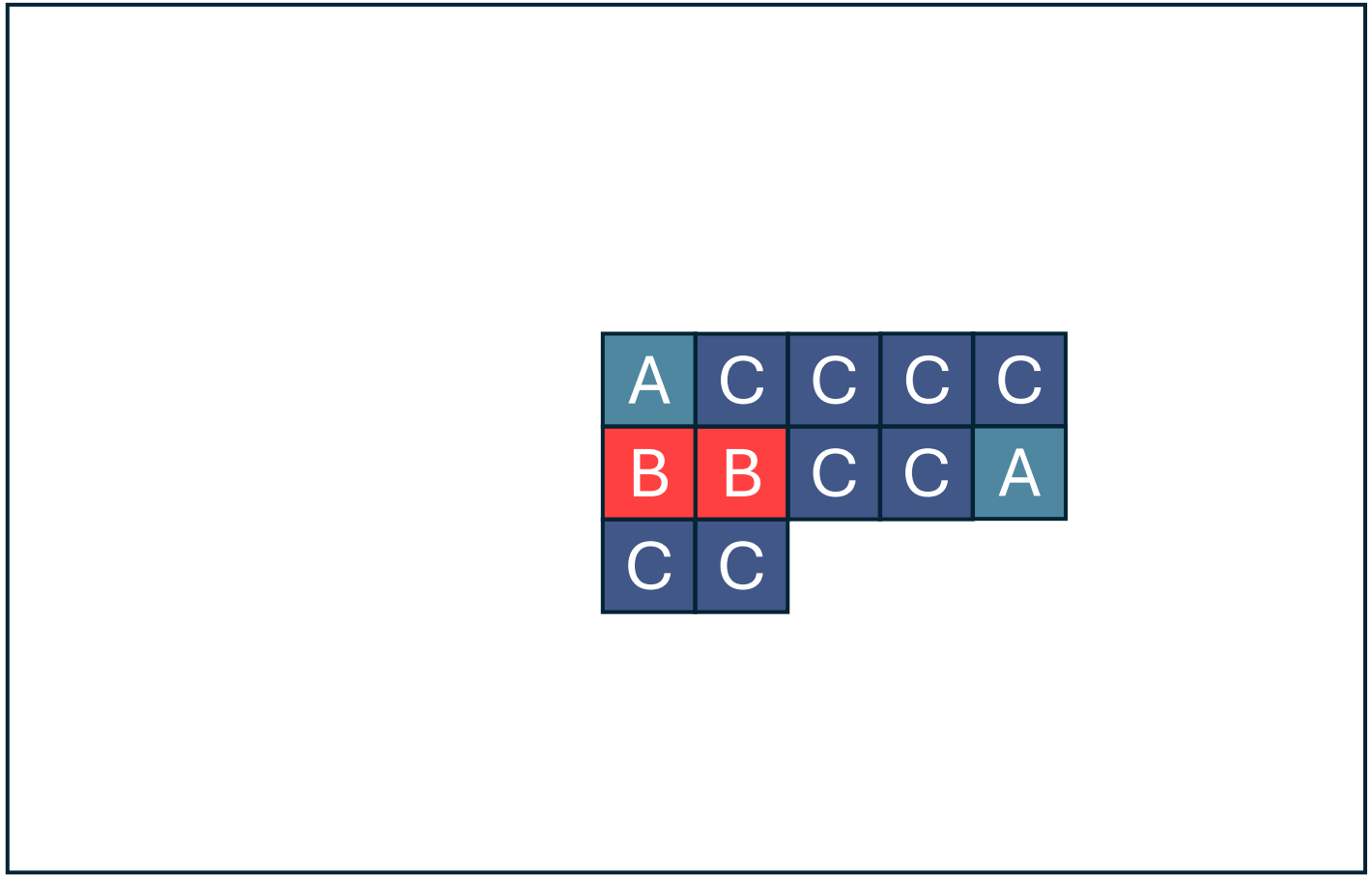


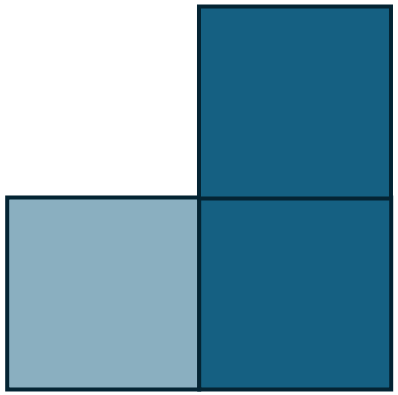
Our Contribution

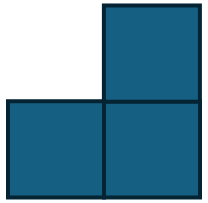
	aTAM	2HAM	STAM	Seeded TA
Description	- Single tile attachments - No detachments - Fixed glues	- Up to 2 assembly attachments - No detachments - Fixed glues	- Up to 2 assembly attachments - Detachments - Glues turn on/off	- **Single tile attachments** - **No detachments** - **Transitions**
Results	- No fractals weakly buildable (temp 1) - Some fractals not strictly buildable (temp > 1)	- Finitely assemble larger class of shapes - Still some non-buildable fractals	- All fractals strictly buildable - Without detachment: some impossible fractals	- **Fractals with generators that have a Ham-path are strictly buildable (temp 1)**

The Construction

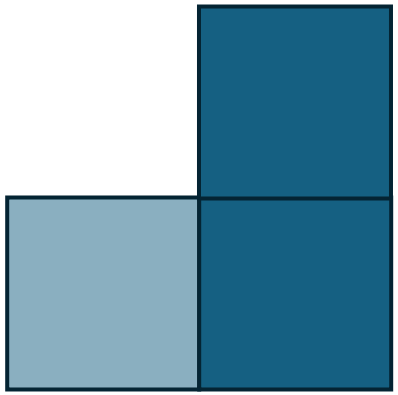
How did we do it?

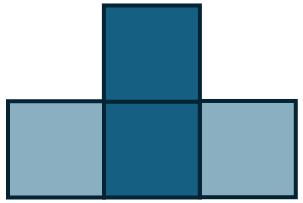
High Level Idea



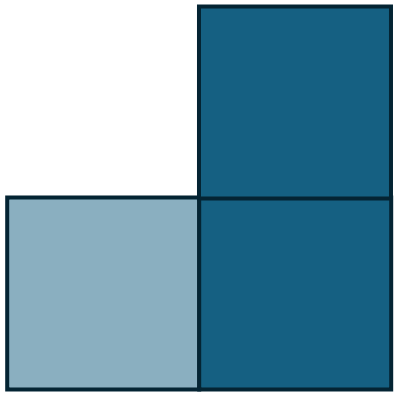


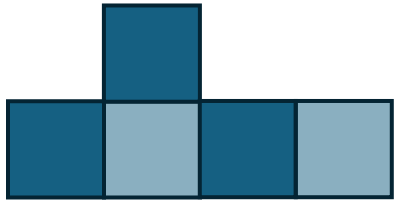
High Level Idea



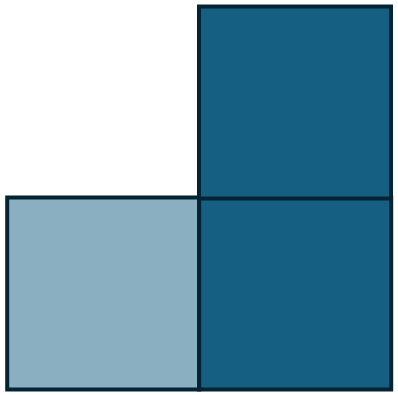


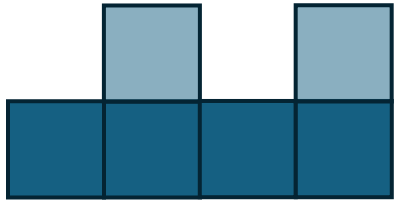
High Level Idea



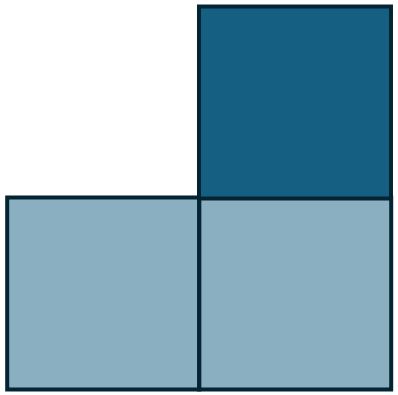


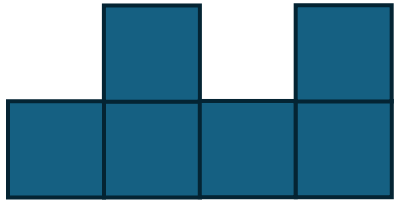
High Level Idea



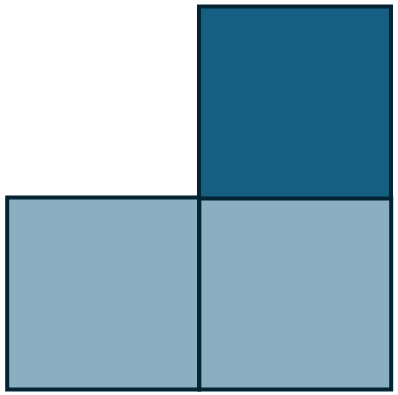


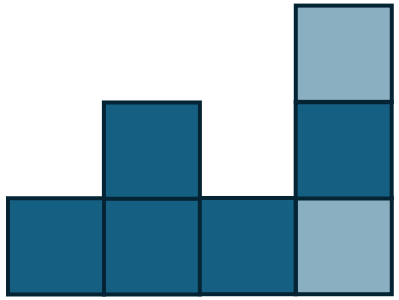
High Level Idea



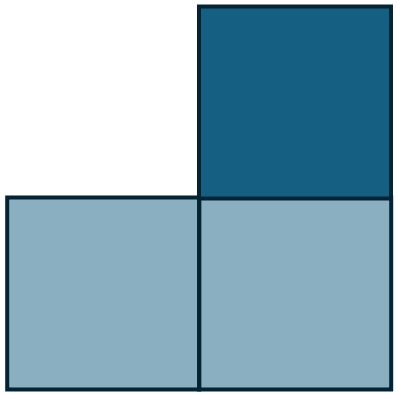


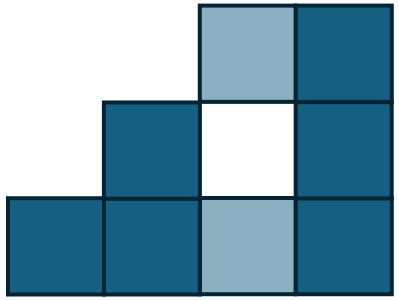
High Level Idea



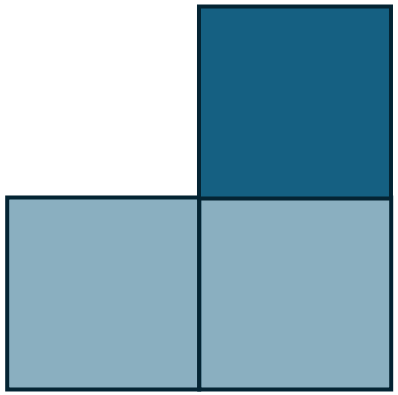


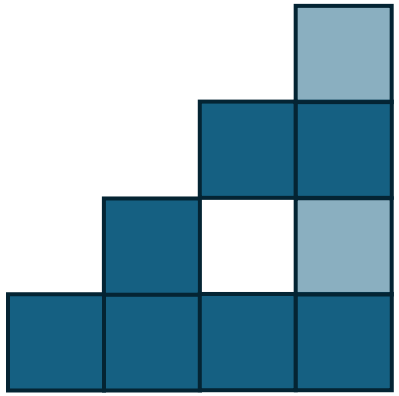
High Level Idea



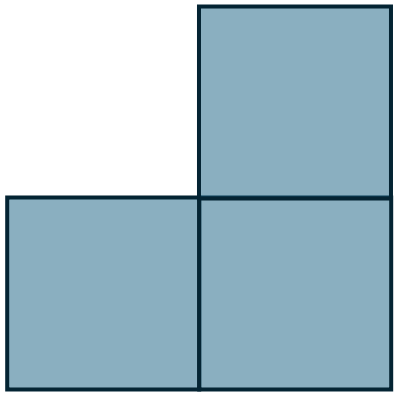


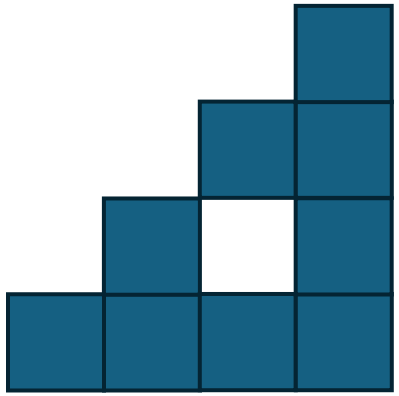
High Level Idea



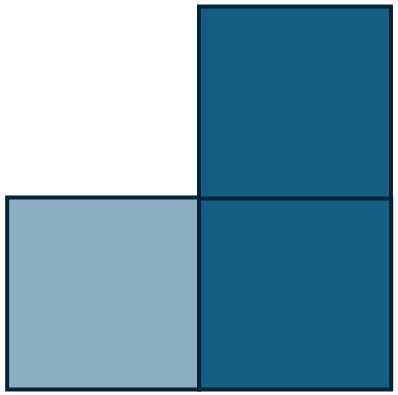


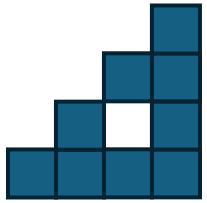
High Level Idea





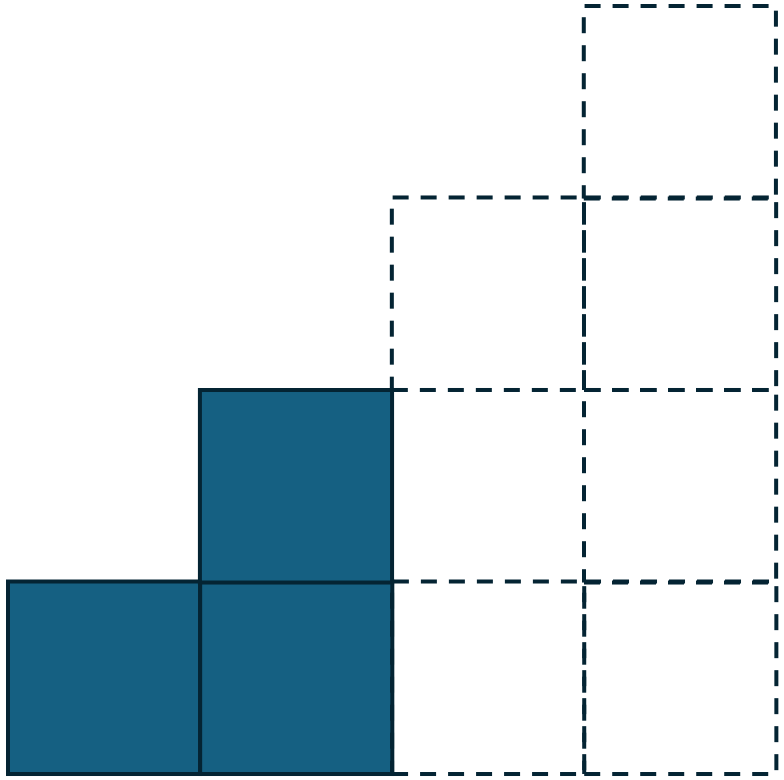
High Level Idea





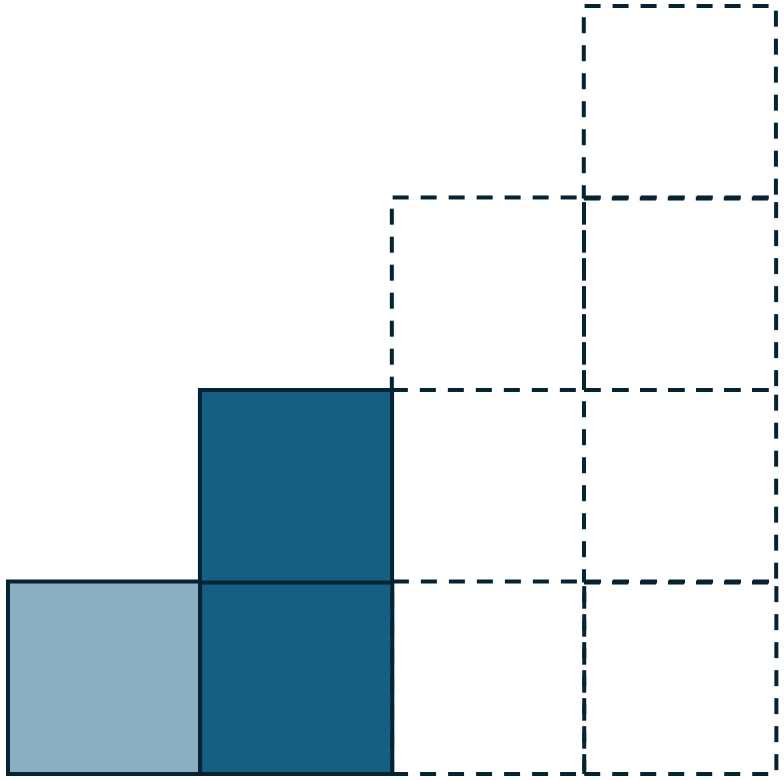
Local Information

1. State of tile



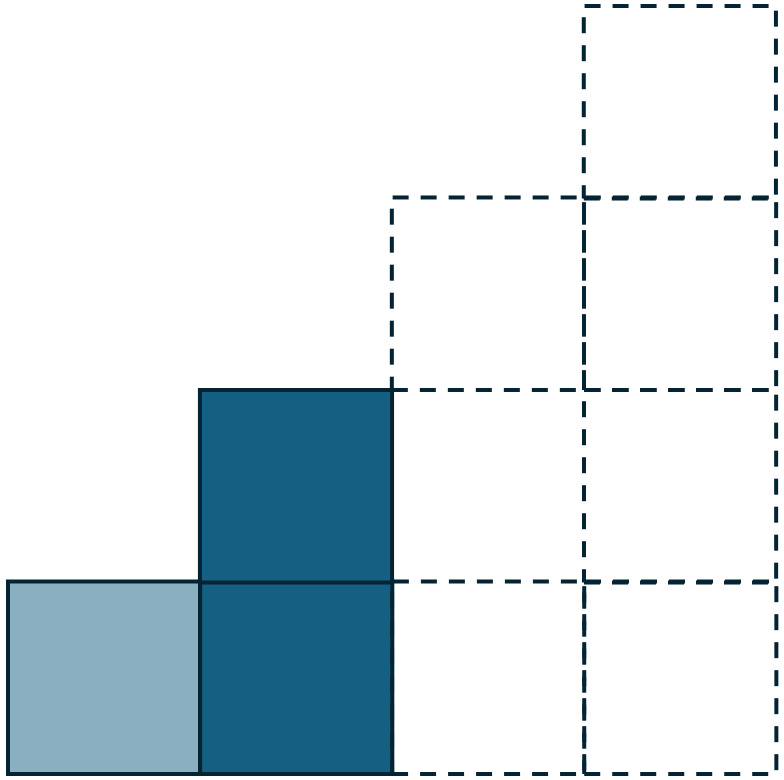
Local Information

1. State of tile



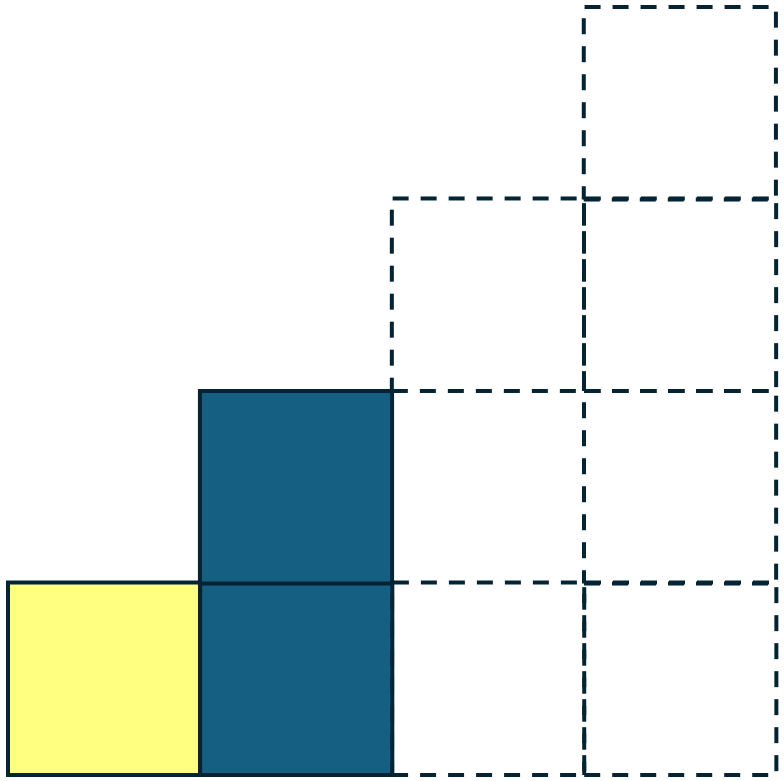
Local Information

1. State of tile
    - Producing



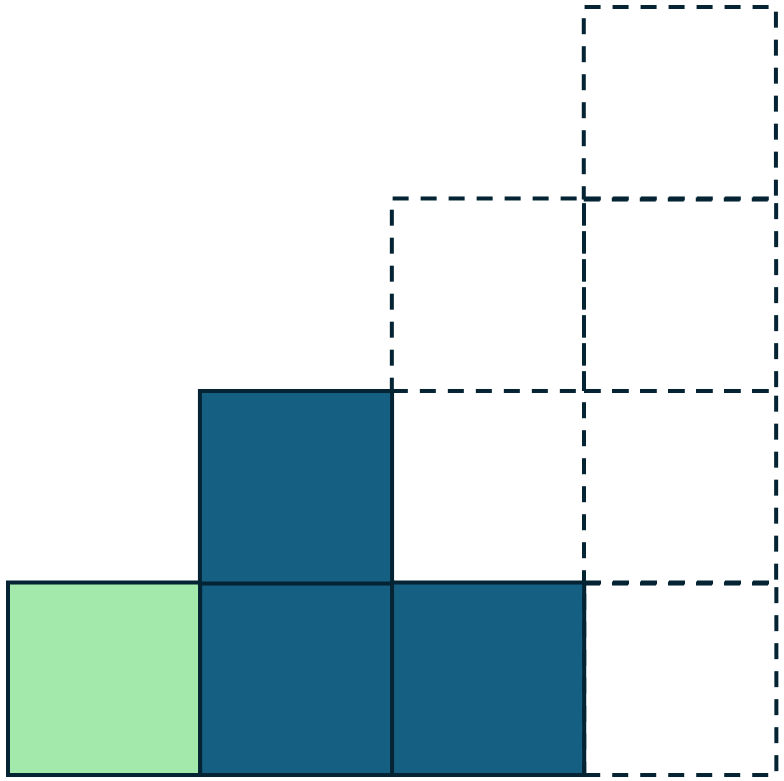
Local Information

1. State of tile
    - Producing
    - Waiting



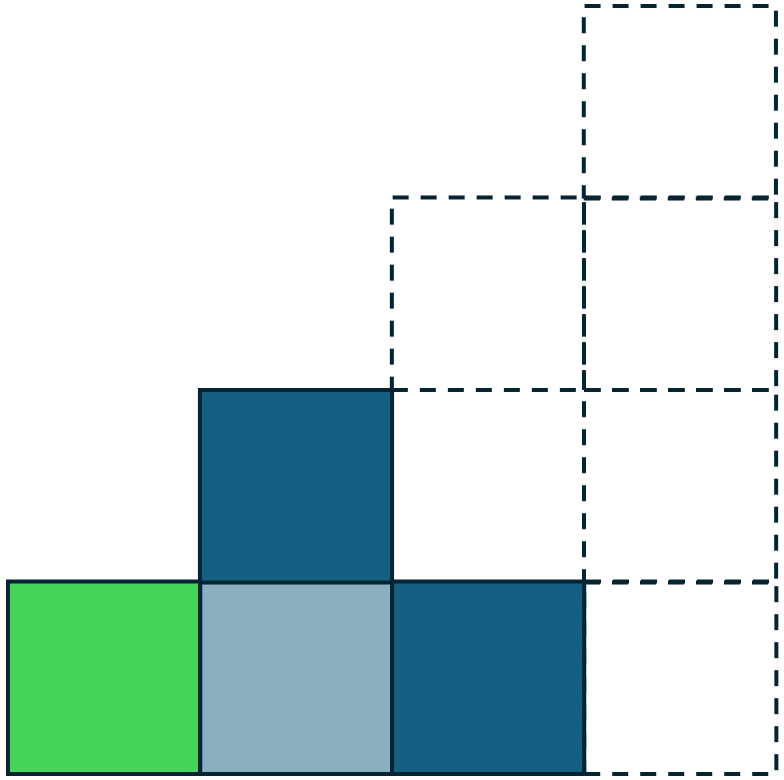
Local Information

1. State of tile
    - Producing
    - Waiting
    - Finished



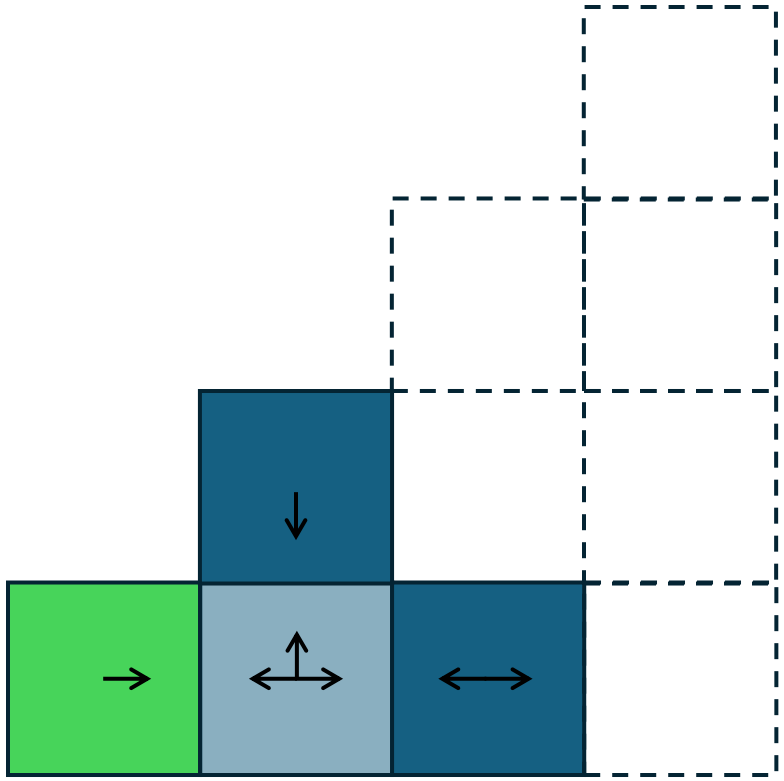
Local Information

1. State of tile
2. Info from neighbors



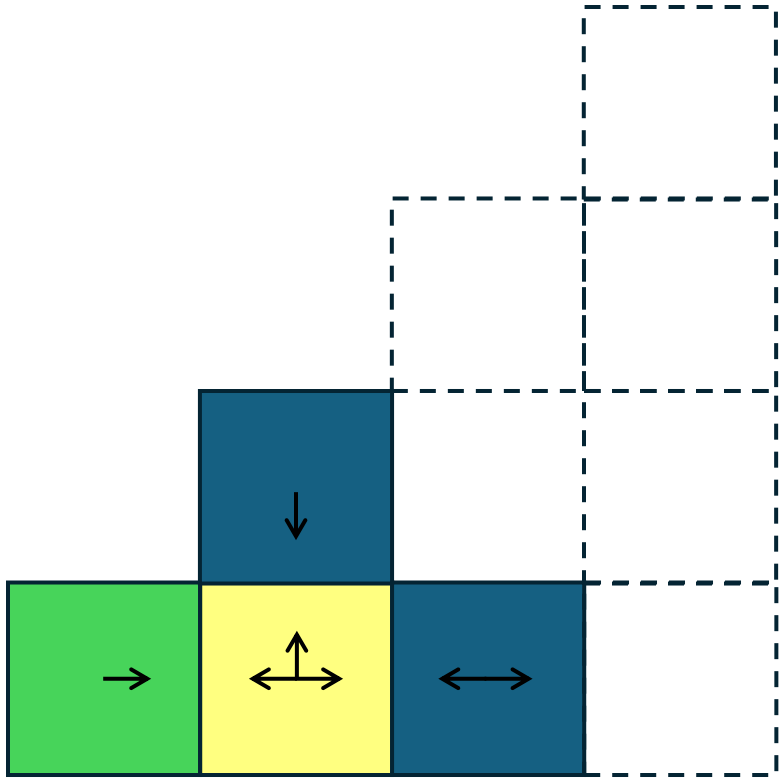
Local Information

1. State of tile
2. Info from neighbors
    - Who are they?
    - State?
    - Tile being placed?



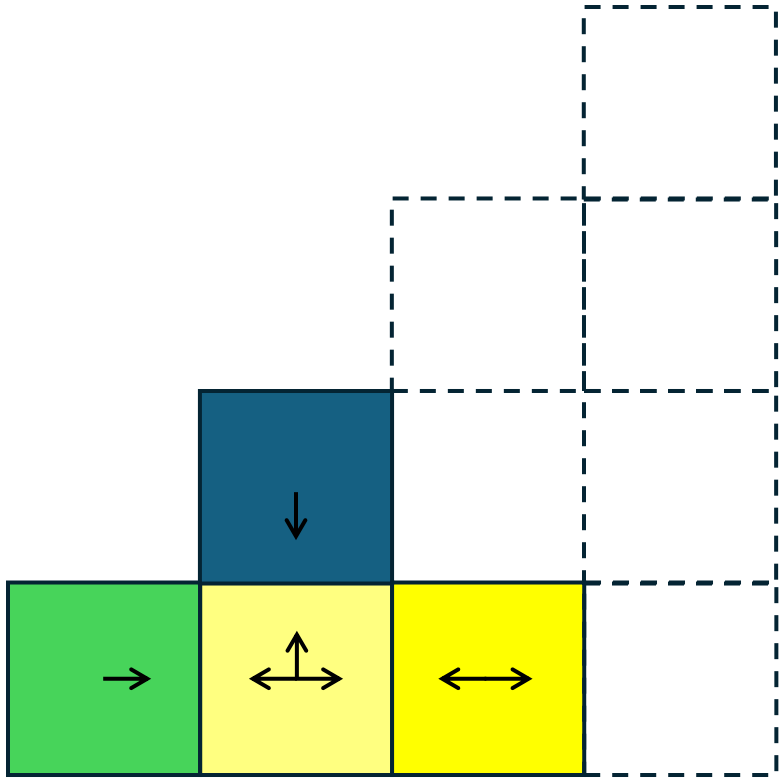
Local Information

1. State of tile
2. Info from neighbors
    - Who are they?
    - State?
    - Tile being placed?



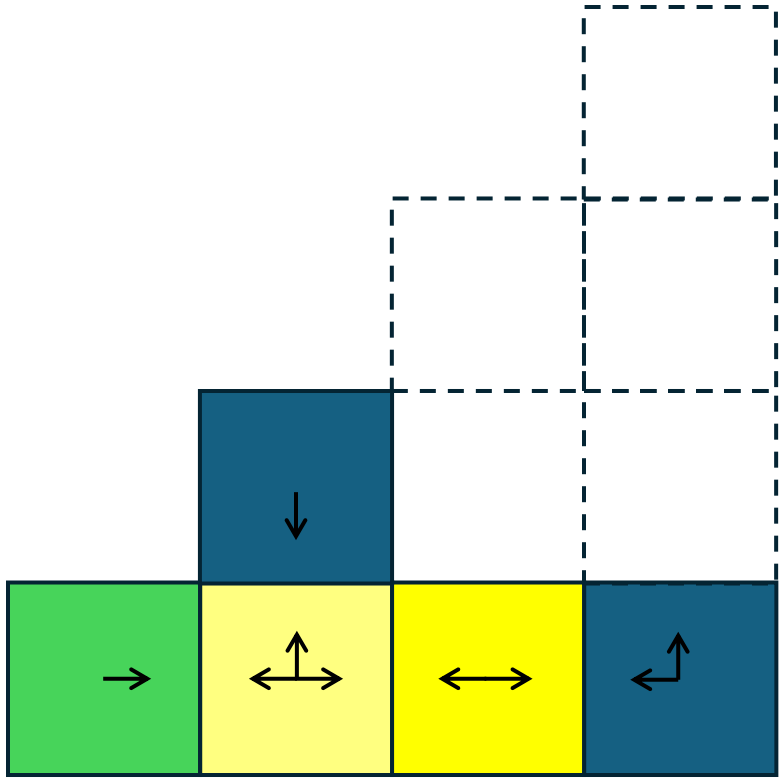
Local Information

1. State of tile
2. Info from neighbors
    - Who are they?
    - State?
    - Tile being placed?



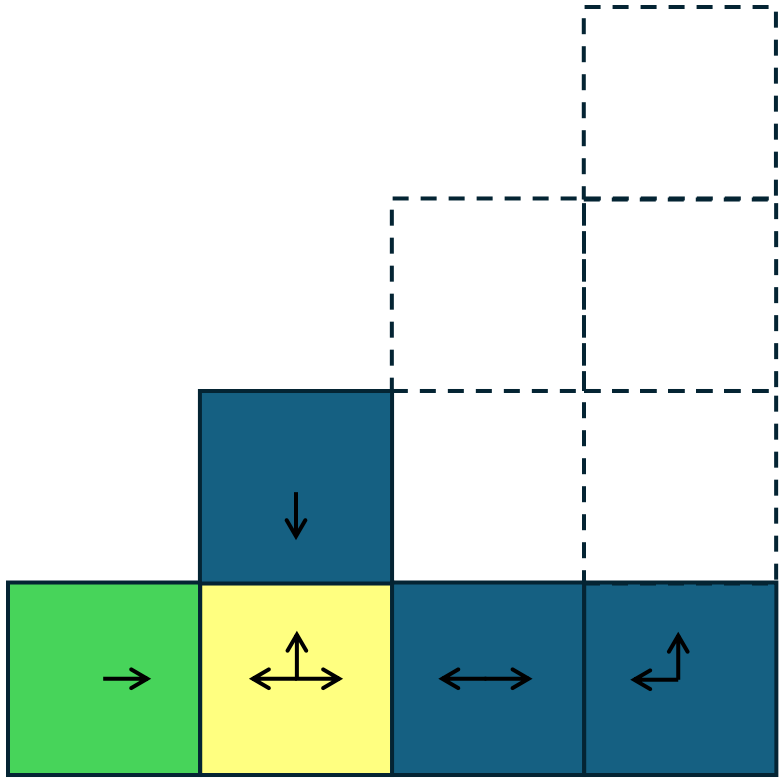
Local Information

1. State of tile
2. Info from neighbors
    - Who are they?
    - State?
    - Tile being placed?



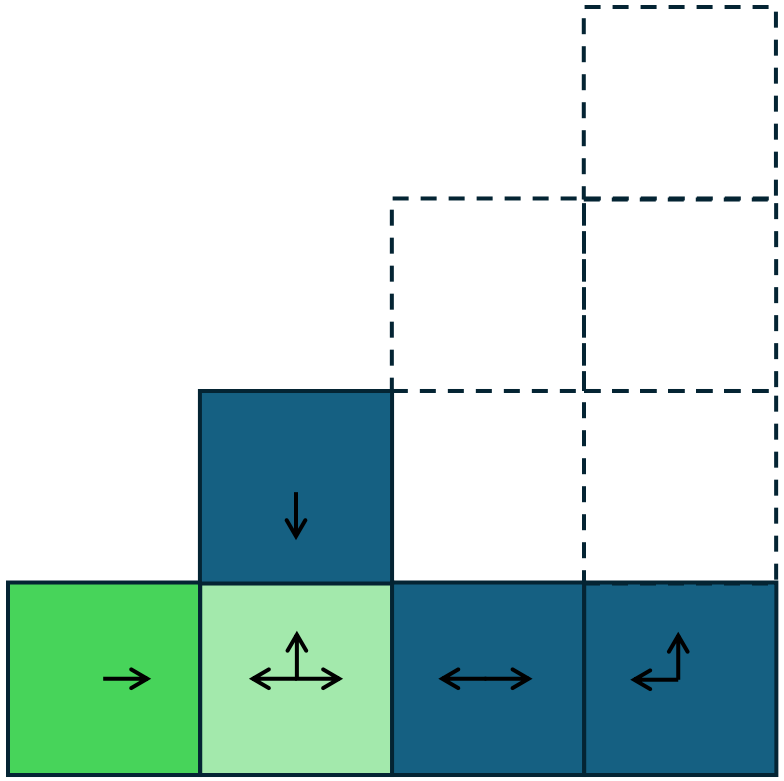
Local Information

1. State of tile
2. Info from neighbors
    - Who are they?
    - State?
    - Tile being placed?



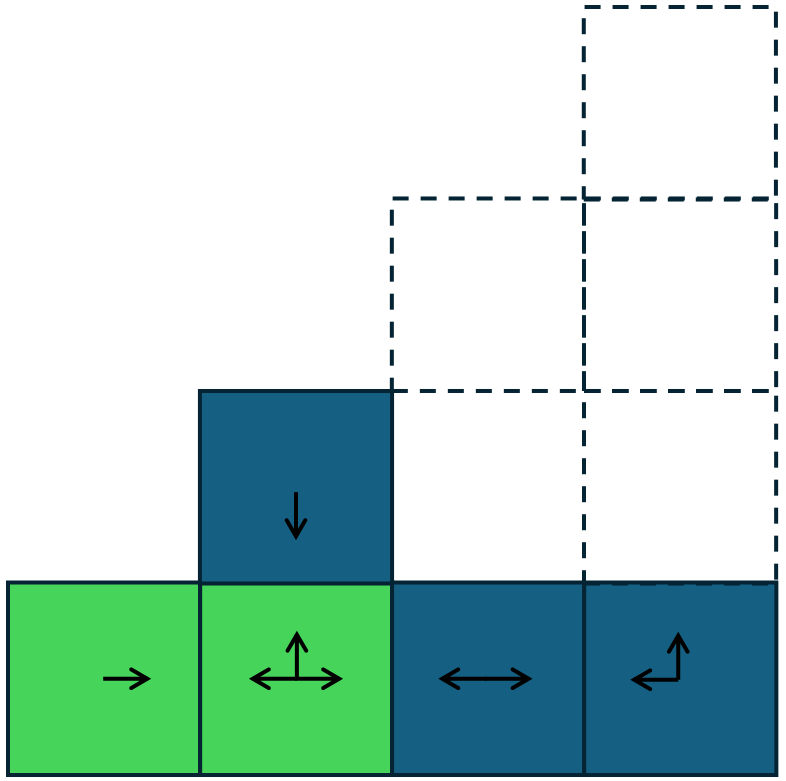
Local Information

1. State of tile
2. Info from neighbors
    - Who are they?
    - State?
    - Tile being placed?



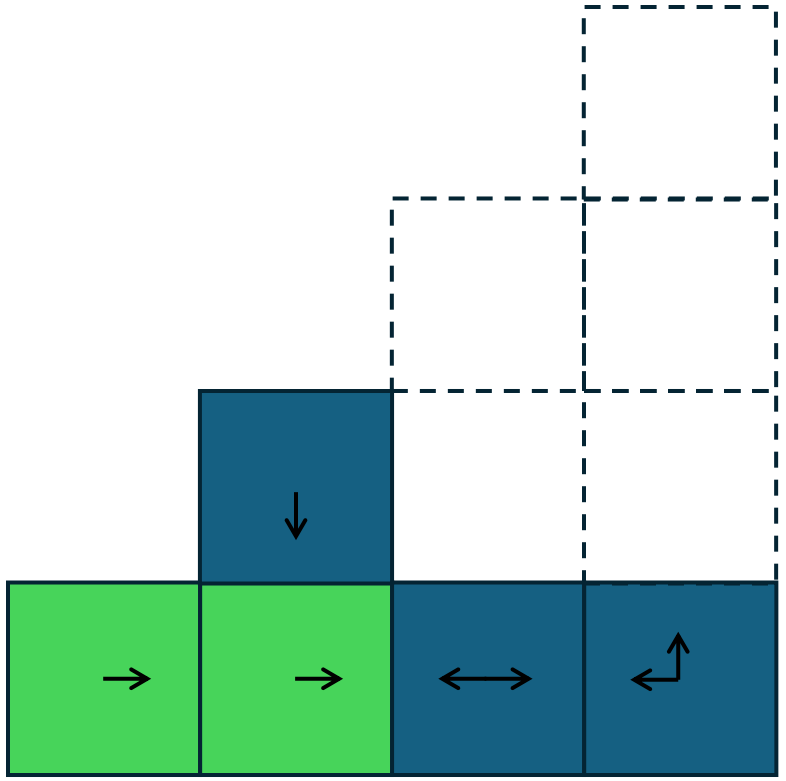
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal



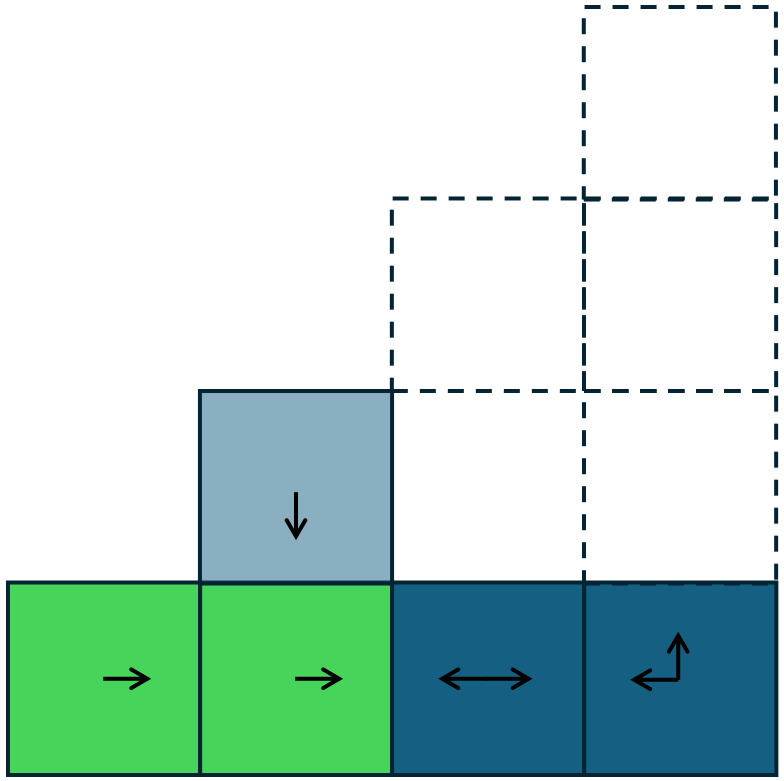
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal



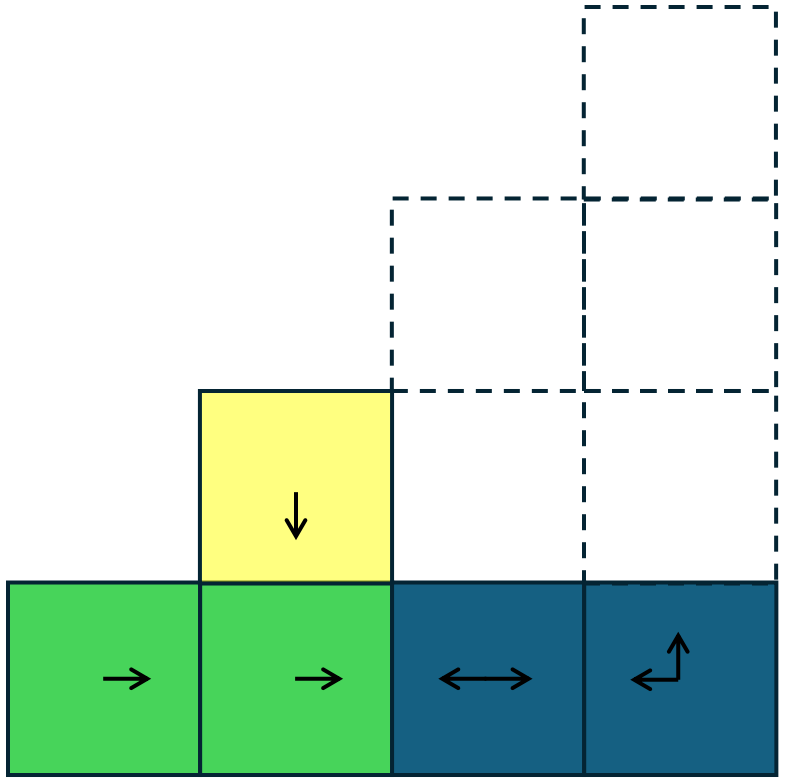
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal



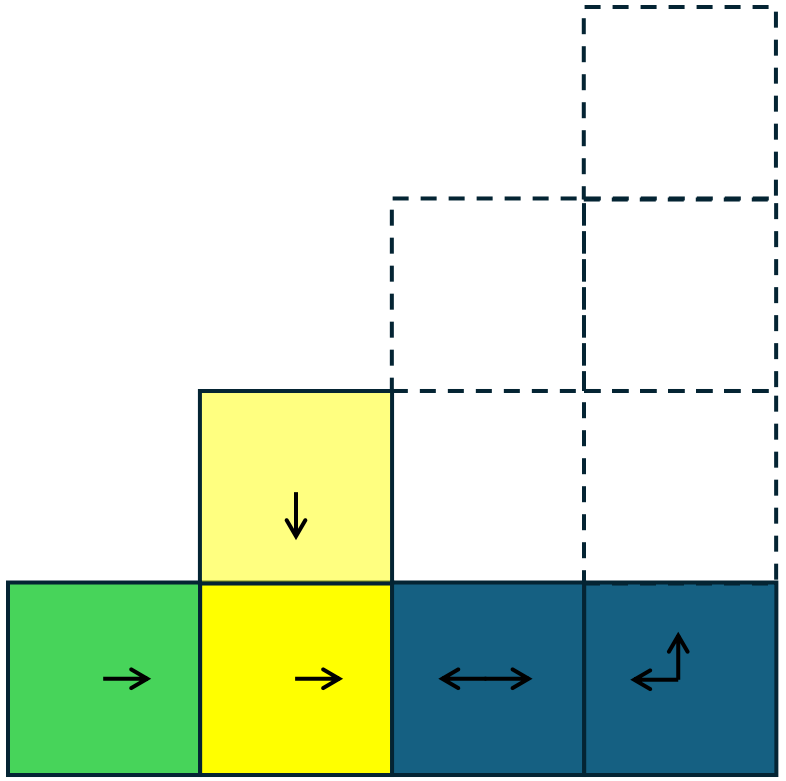
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal



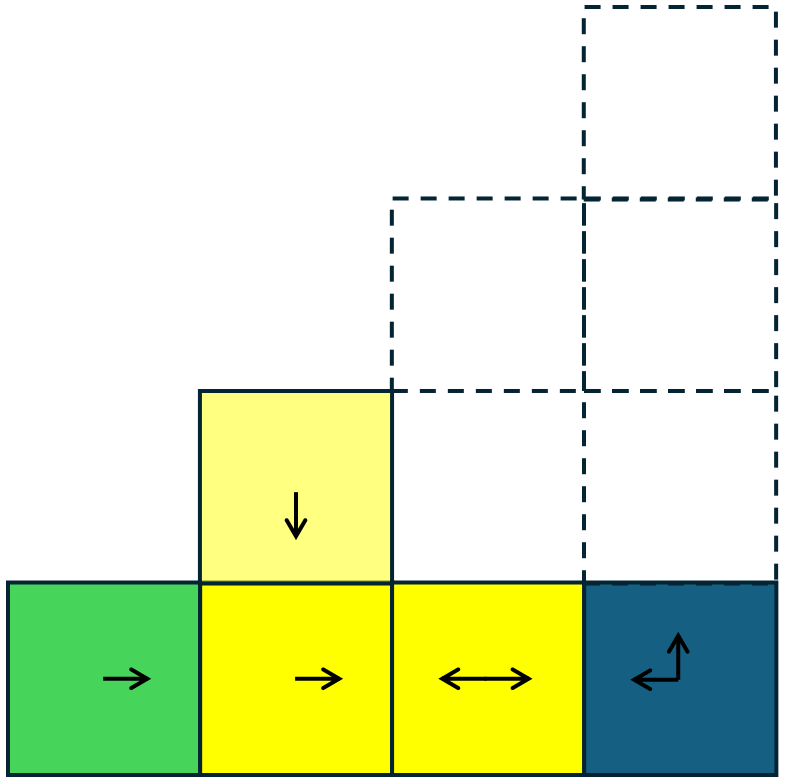
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal



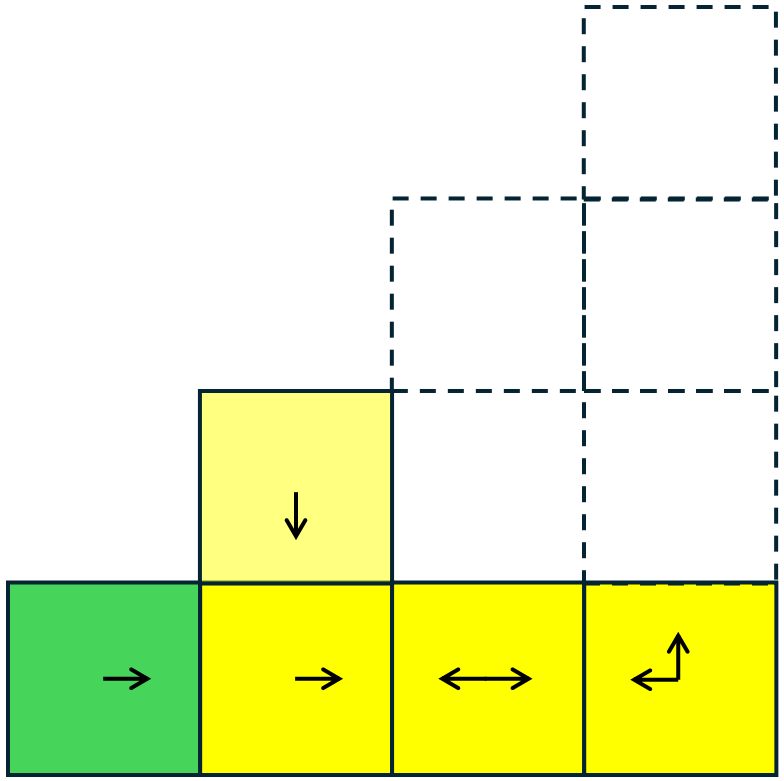
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal



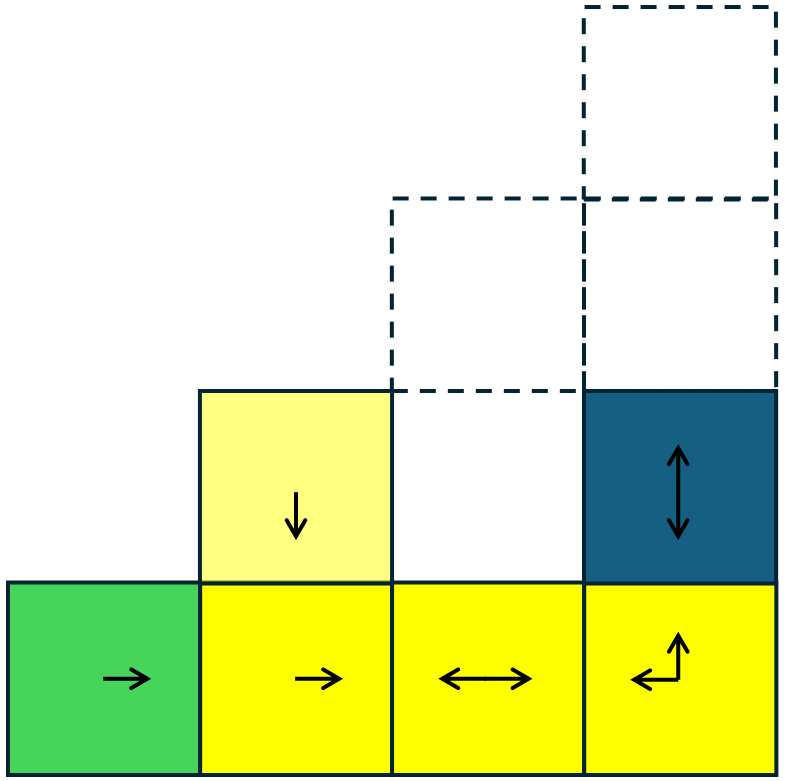
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal



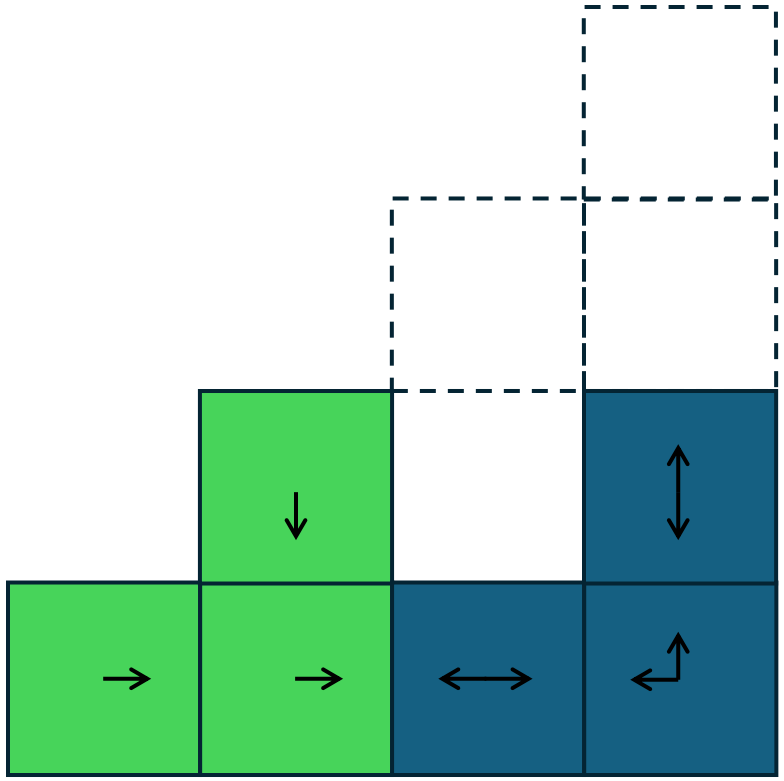
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal



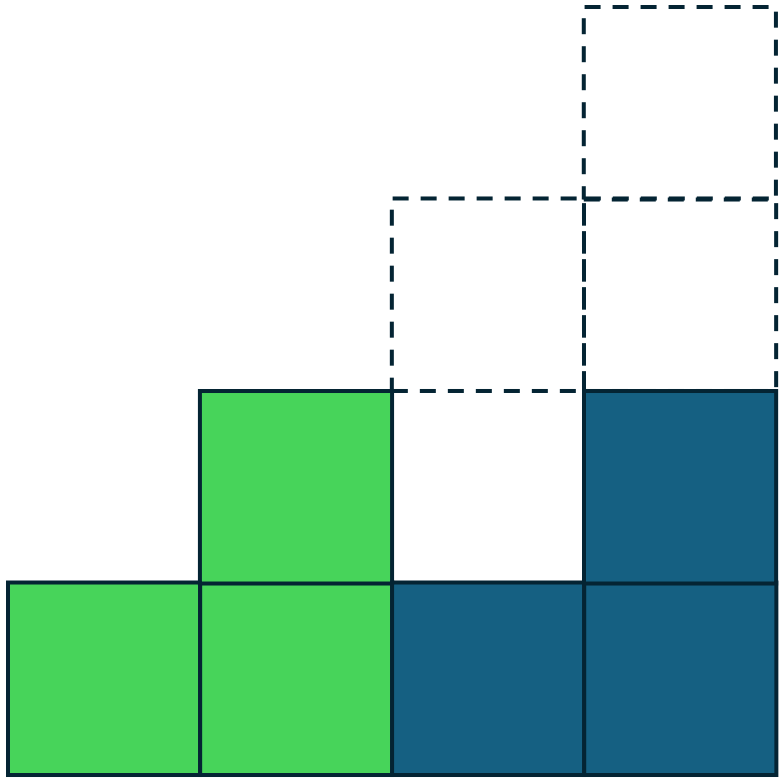
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal



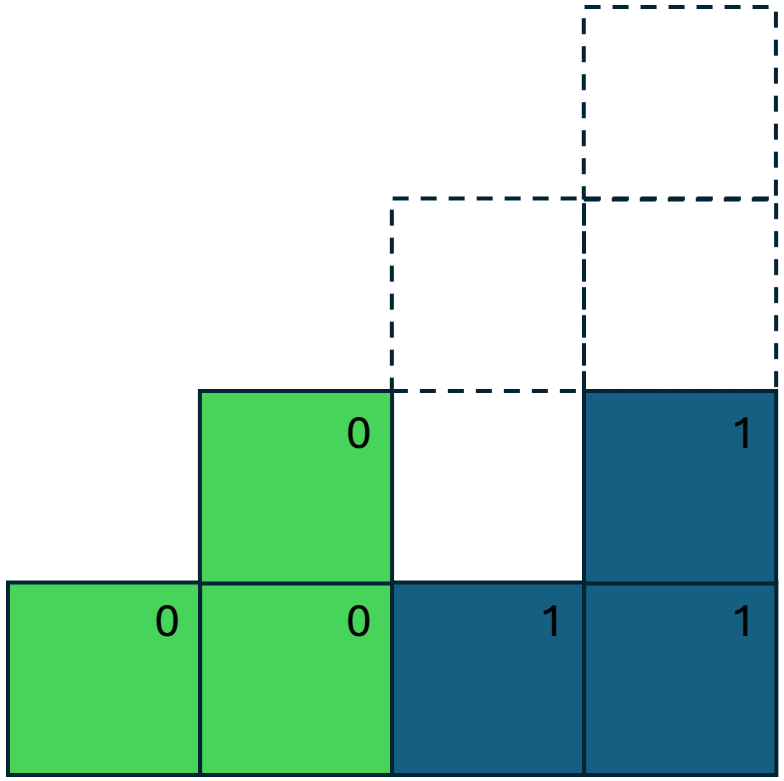
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal
4. Step



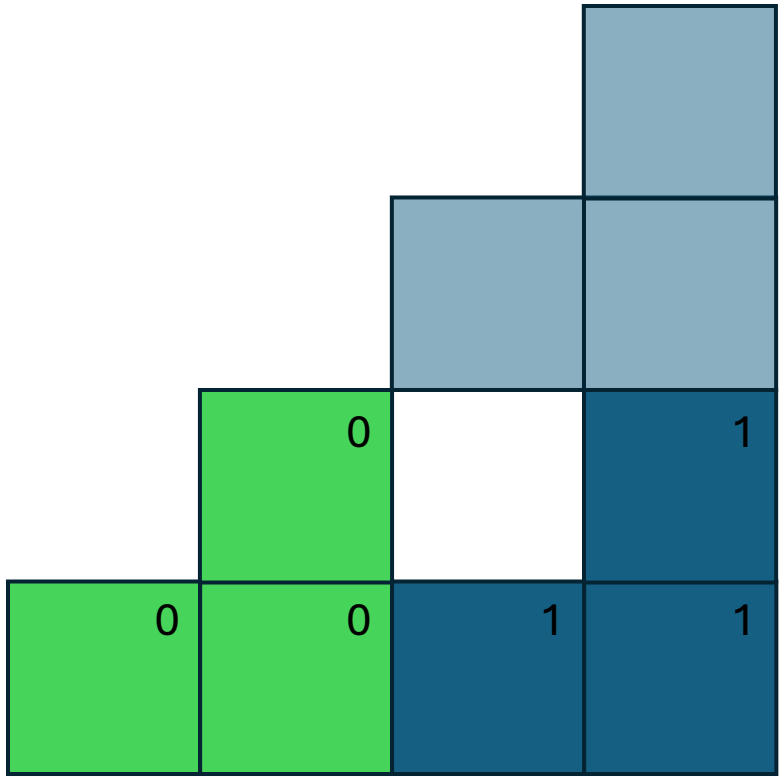
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal
4. Step



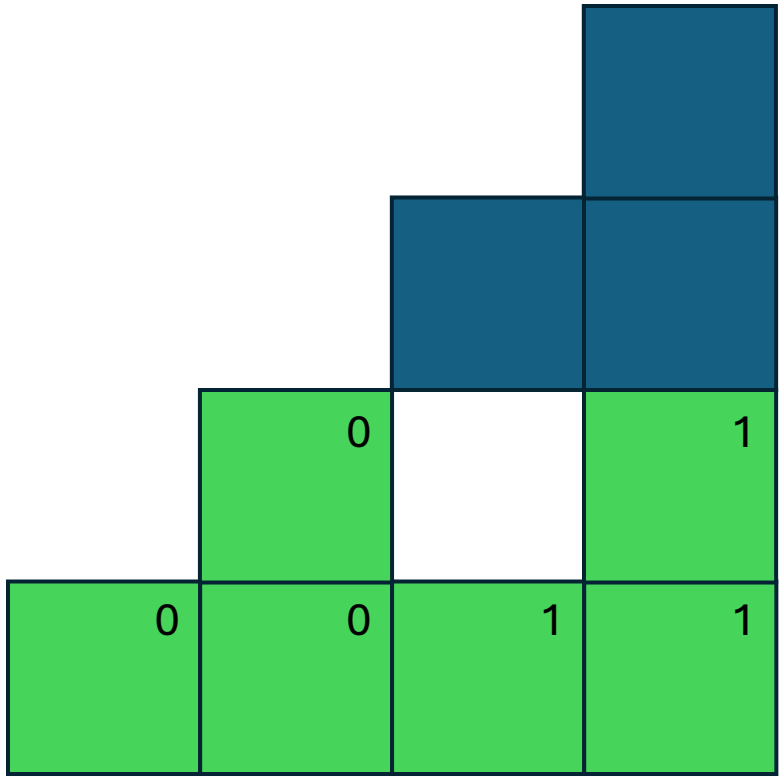
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal
4. Step



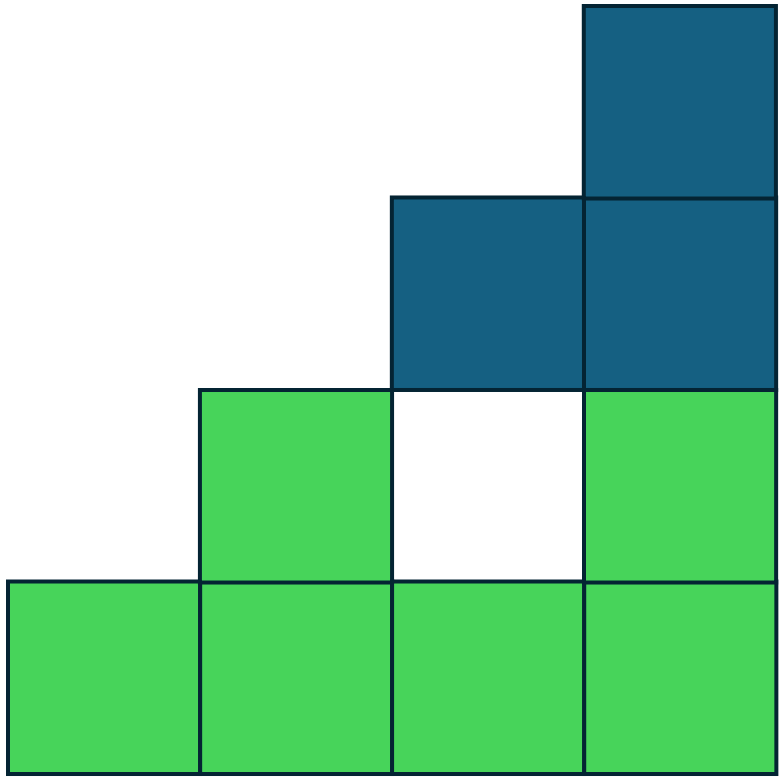
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal
4. Step



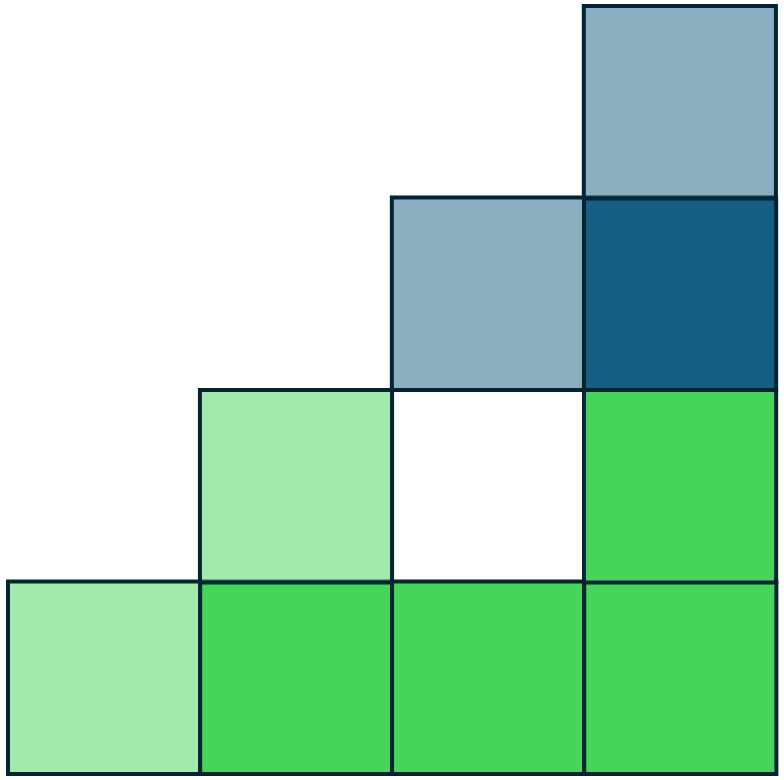
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal
4. Step
5. Terminal?



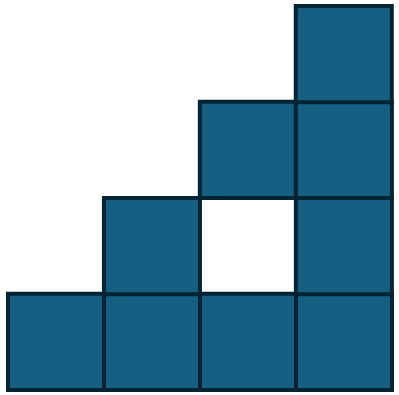
Local Information

1. State of tile
2. Info from neighbors
3. Direction to send signal
4. Step
5. Terminal?



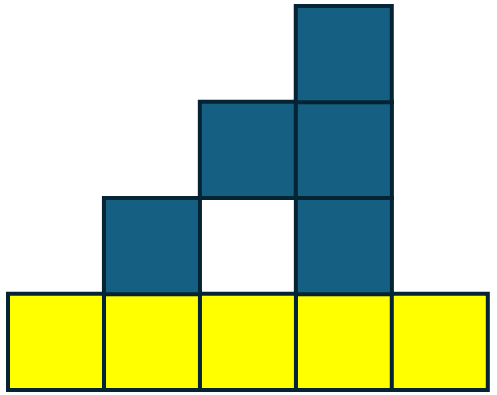
The Process

1. First tile places itself



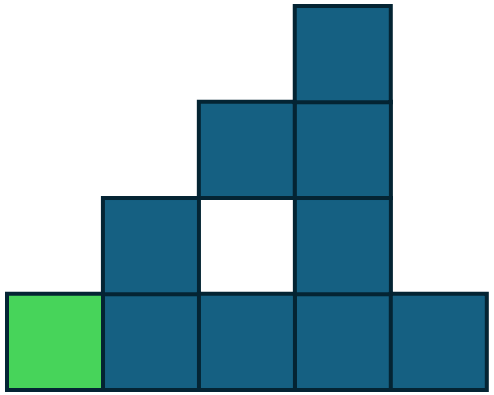
The Process

1. First tile places itself



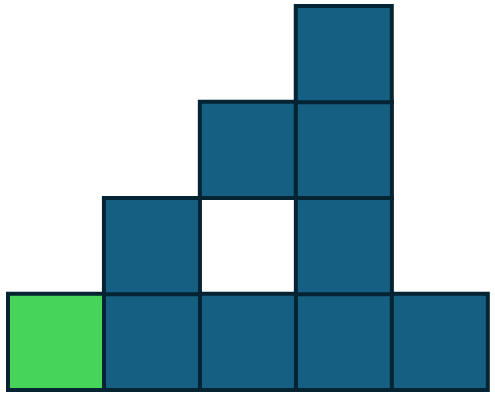
The Process

1. First tile places itself



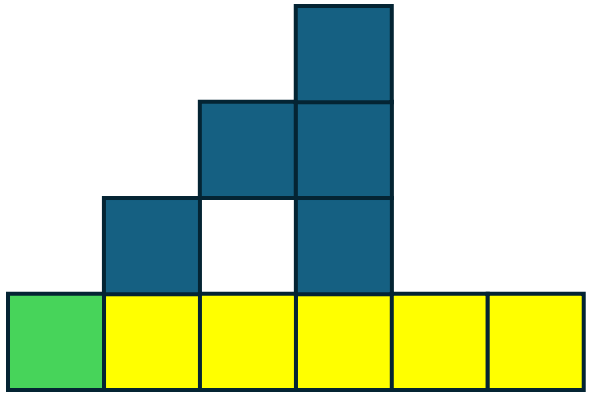
The Process

1. First tile places itself
2. Next tile is chosen, repeats



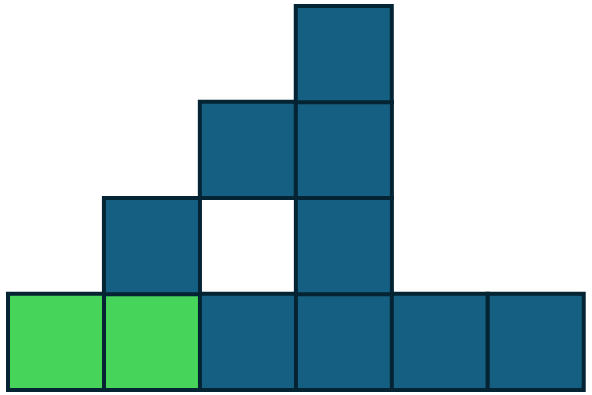
The Process

1. First tile places itself
2. Next tile is chosen, repeats



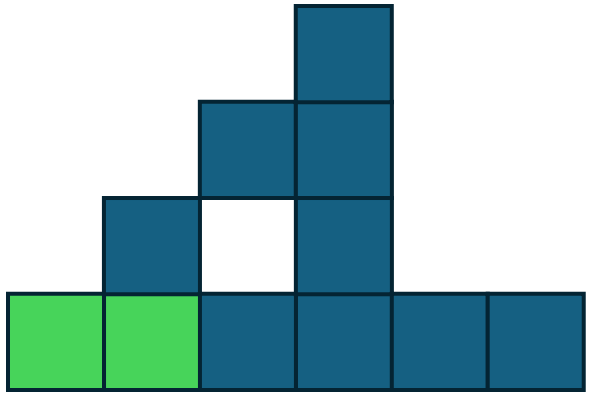
The Process

1. First tile places itself
2. Next tile is chosen, repeats



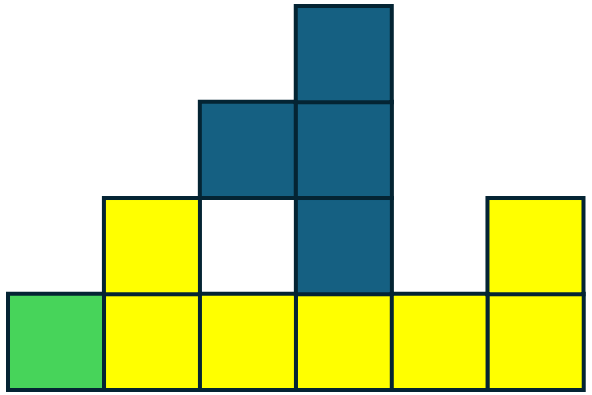
The Process

1. First tile places itself
2. Next tile is chosen, repeats
3. If terminal, a cap is placed



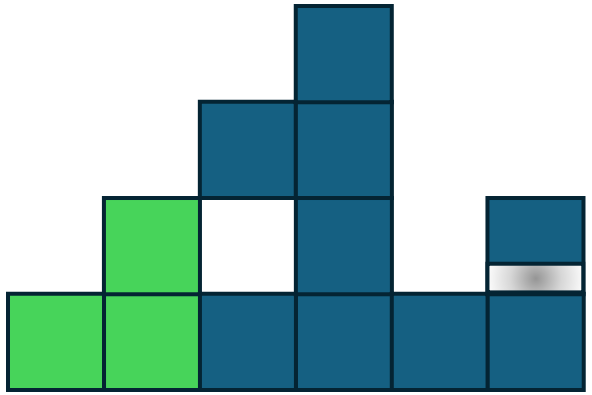
The Process

1. First tile places itself
2. Next tile is chosen, repeats
3. If terminal, a cap is placed



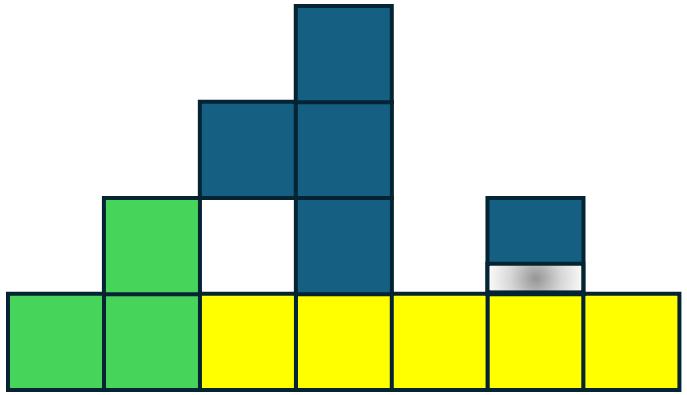
The Process

1. First tile places itself
2. Next tile is chosen, repeats
3. If terminal, a cap is placed



The Process

1. First tile places itself
2. Next tile is chosen, repeats
3. If terminal, a cap is placed



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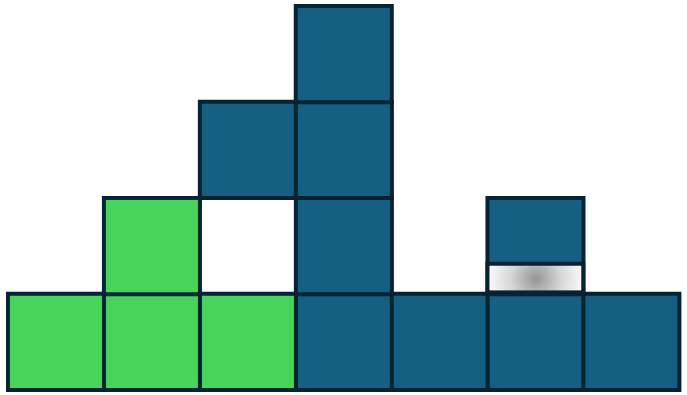
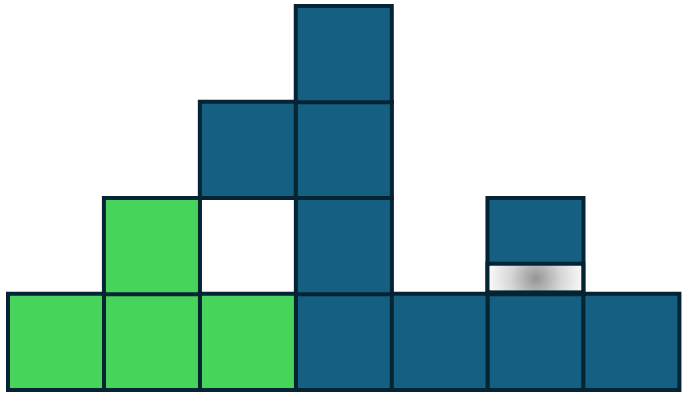


The Process

1. First tile places itself
2. Next tile is chosen, repeats
3. If terminal, a cap is placed
4. Shift caps if needed



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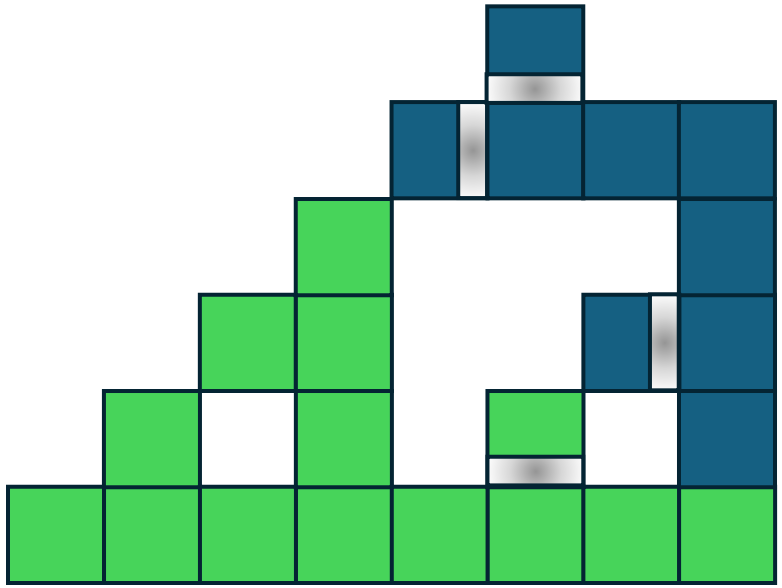

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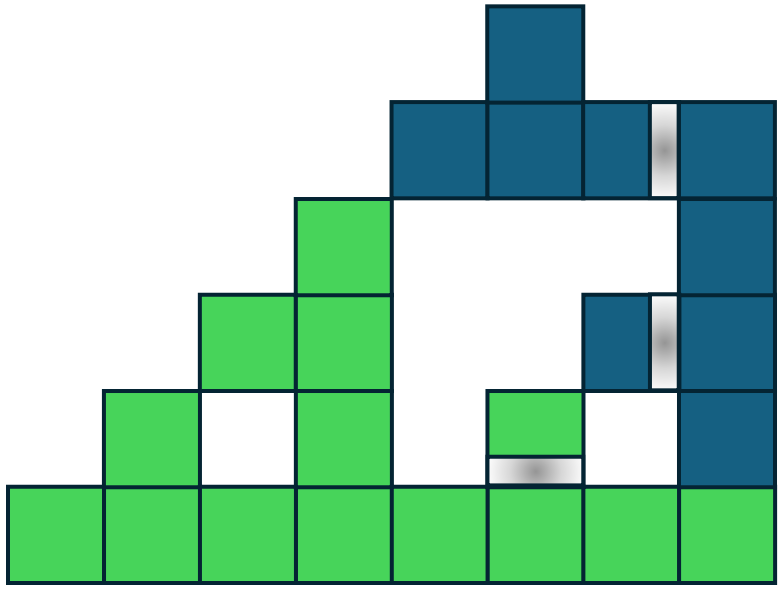
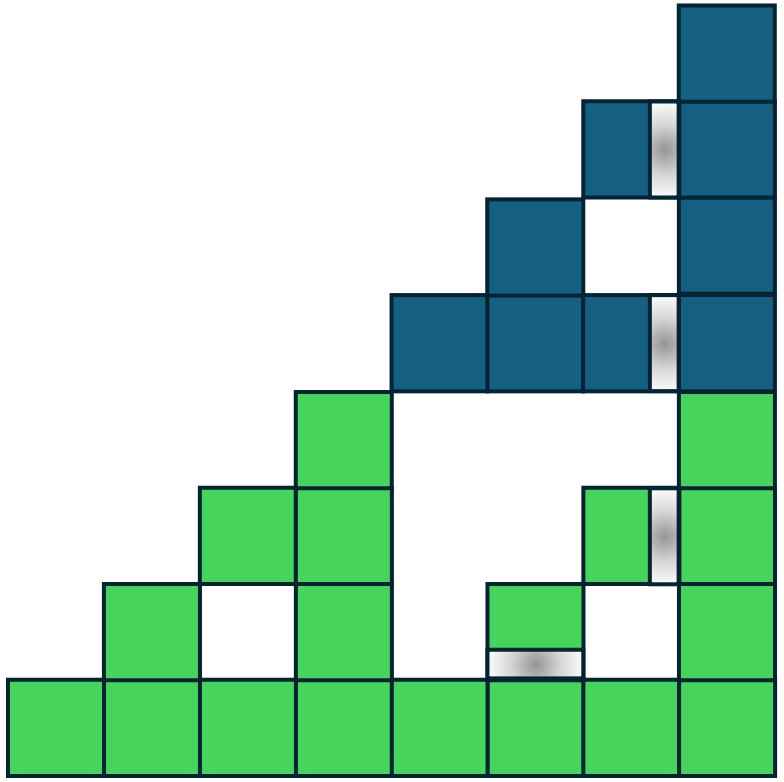


The Process

1. First tile places itself
2. Next tile is chosen, repeats
3. If terminal, a cap is placed
4. Shift caps if needed
5. Reset when complete



The Process

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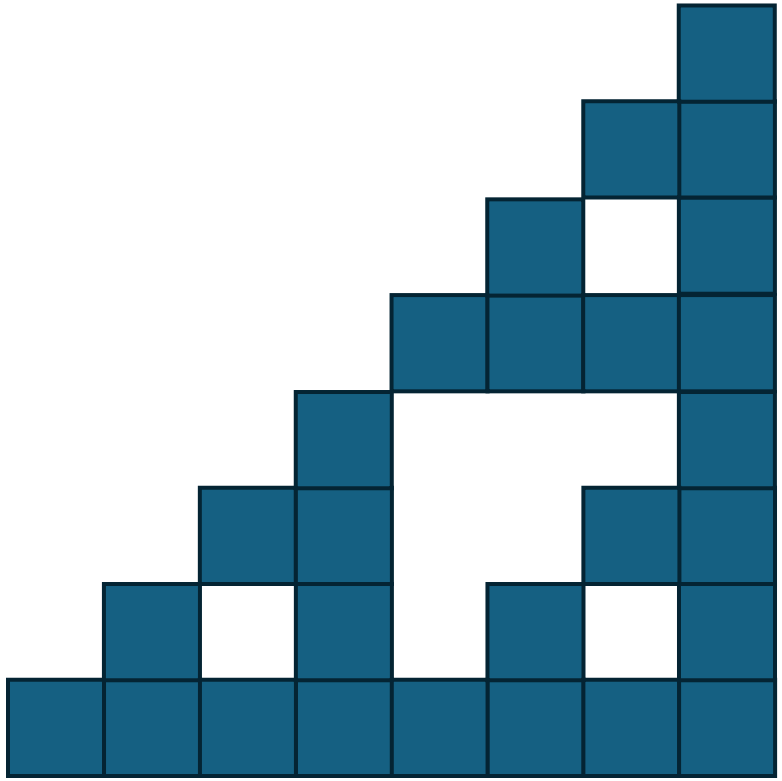


Final Result (Sierpinski Triangle)

Brief Proof Sketch

By definition ($c > 1, X \subset \mathbb{N}^2$), X is a c-discrete s.s. fractal if

$$\exists\, G \subseteq \{0, \dots, c-1\}\, x\, \{0, \dots, c-1\}$$

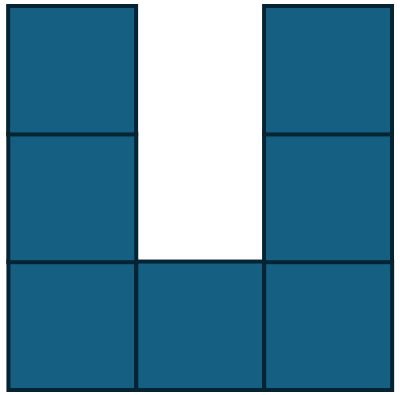
where

$$X = \bigcup_{i=0}^{\infty} X_i$$

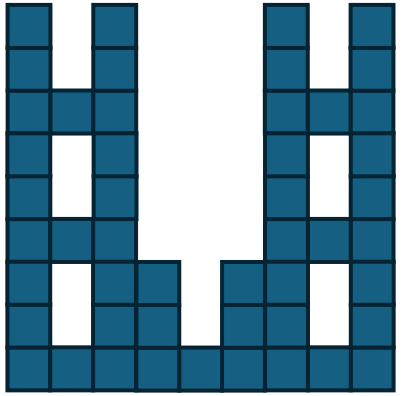
satisfying

$$X_0 = \{(0,0)\},\ X_1 = G,\ X_{i+1} = X_i \cup (X_i + c^i G)$$

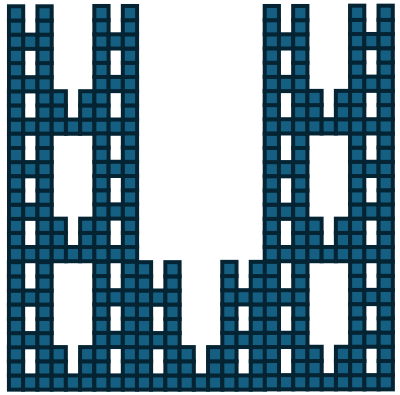
Brief Proof Sketch



$X_1 = G$



X_2



X_3

Our construction follows this!

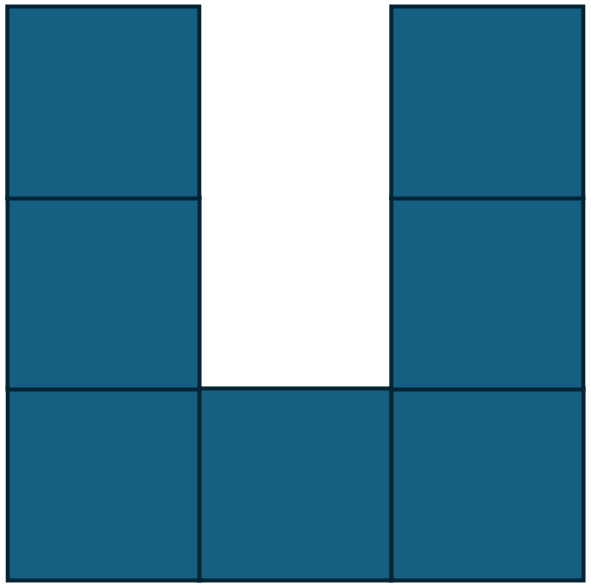
Future Work

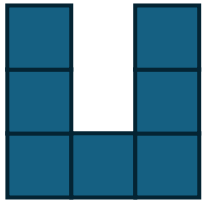
What is left?

Future Work

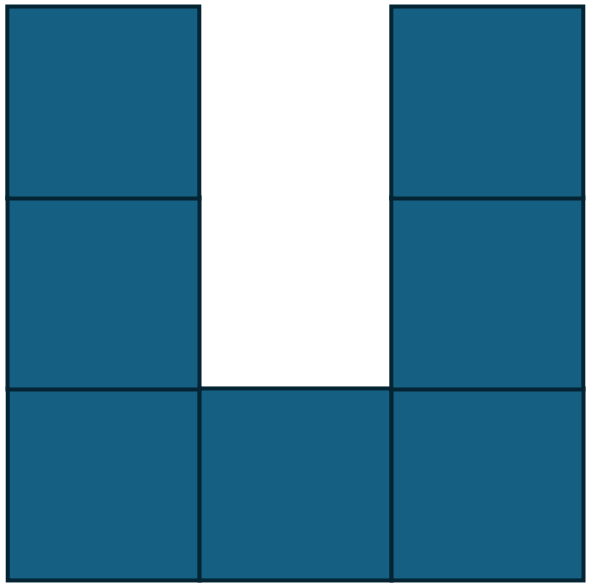
- Does this extend to **ALL** generators?

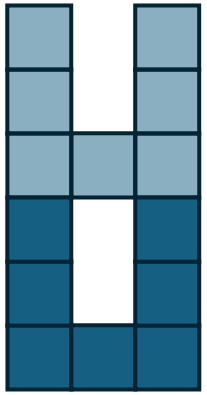
Future Work



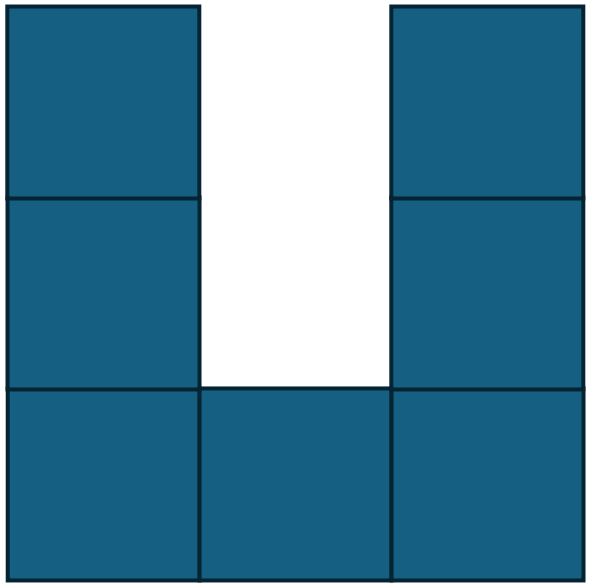


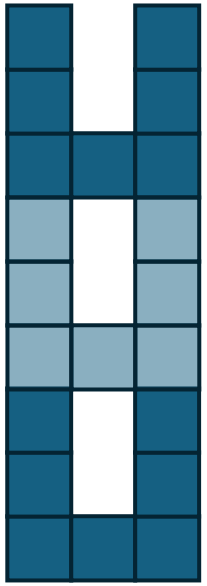
Future Work



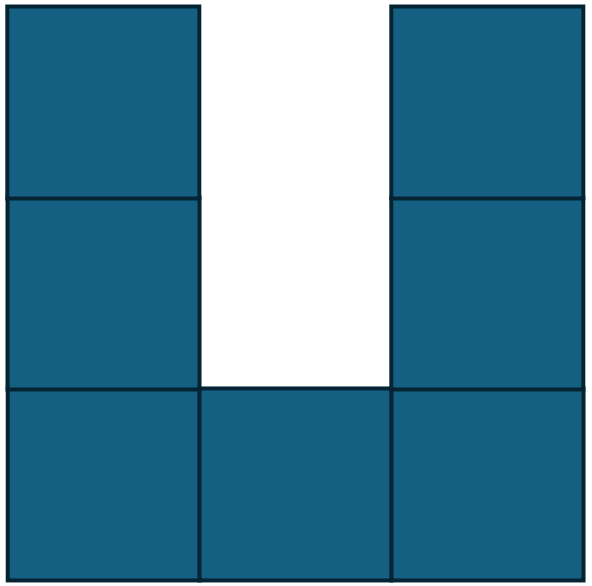


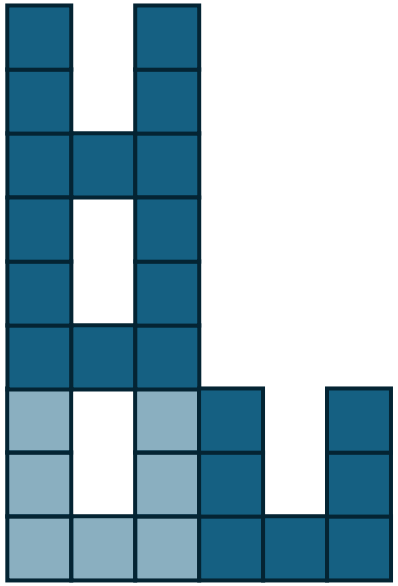
Future Work





Future Work





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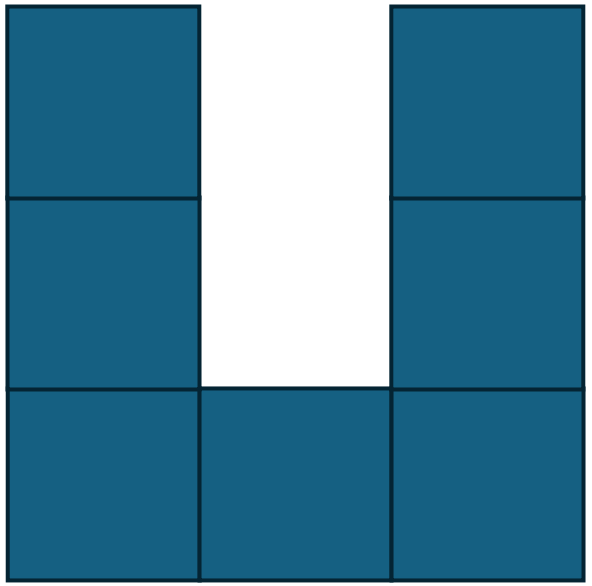
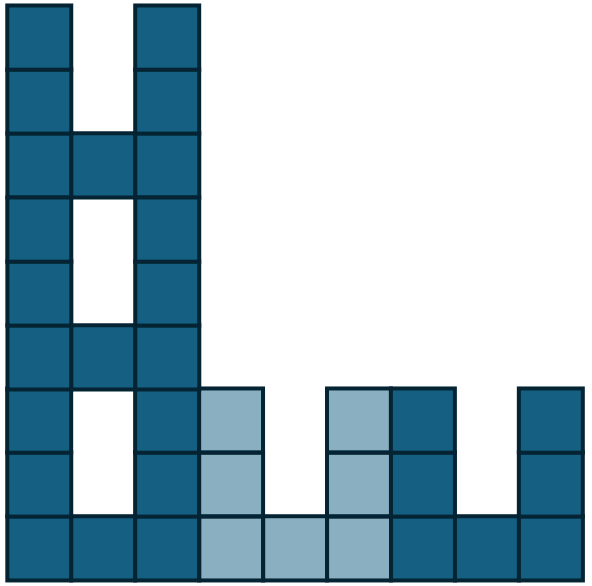
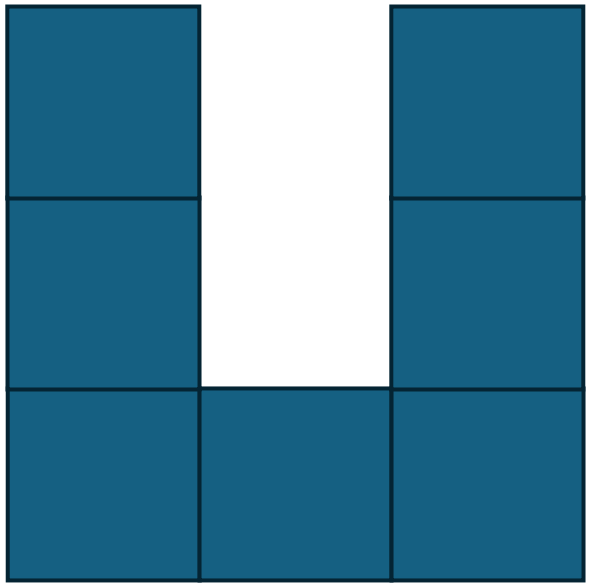
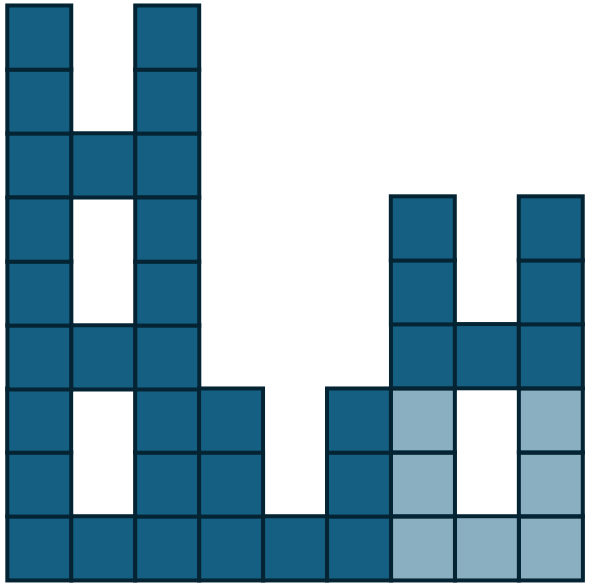


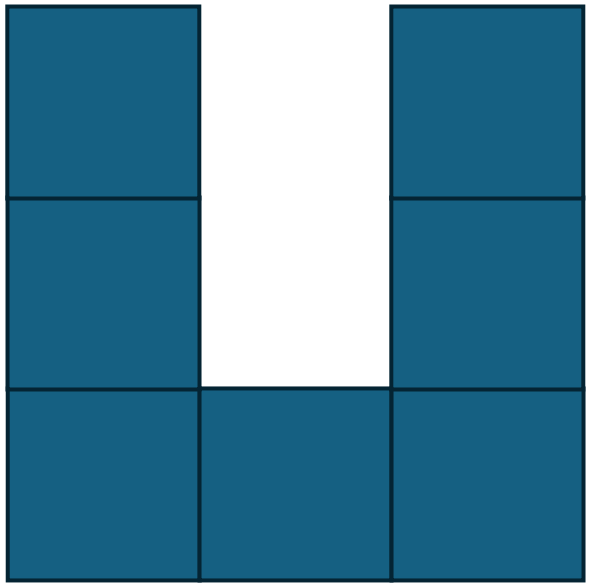


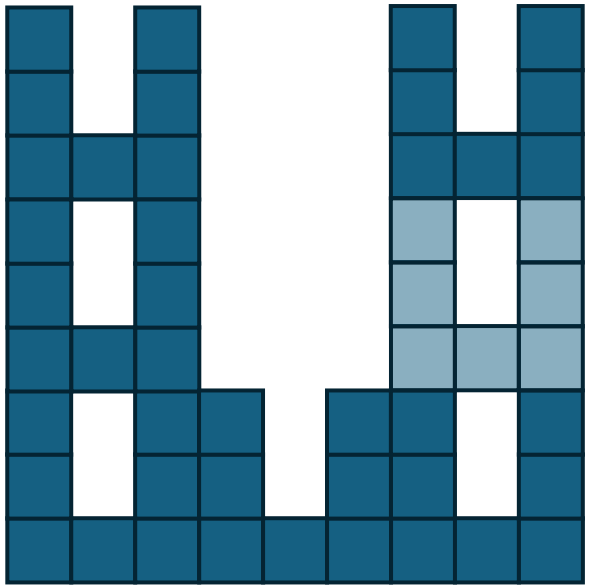
Future Work



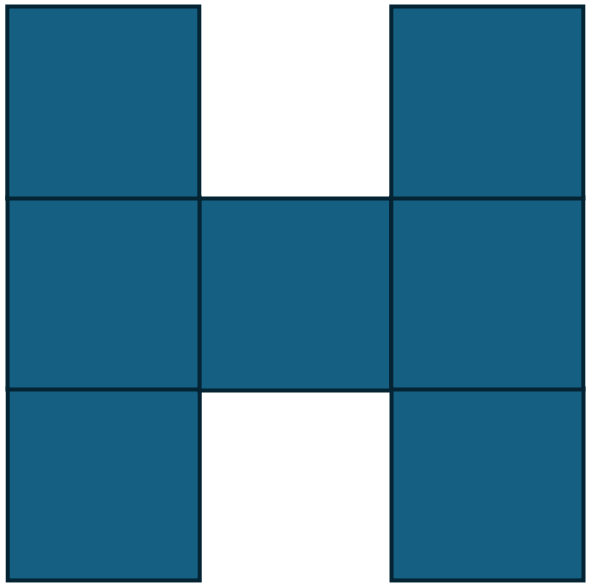


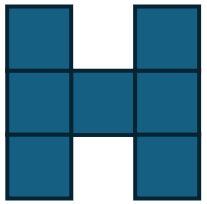
Future Work



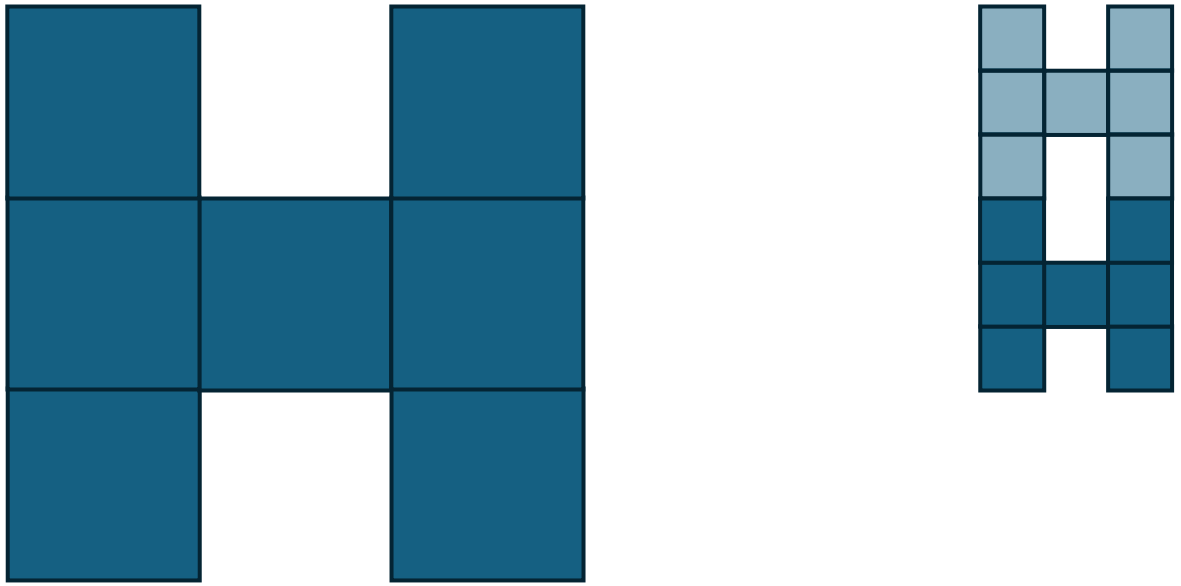


Future Work





Future Work



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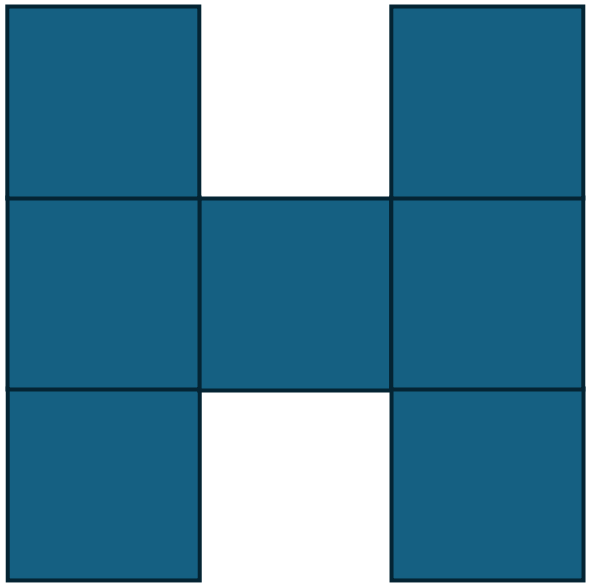

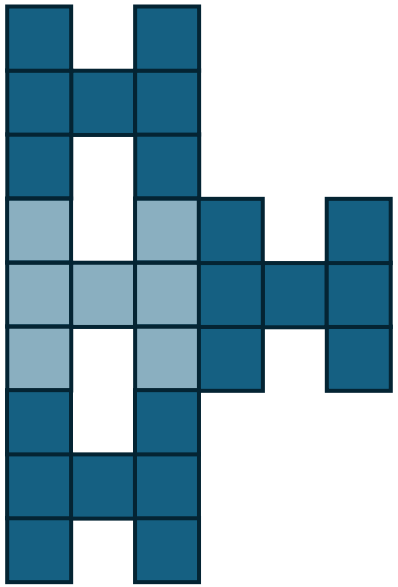

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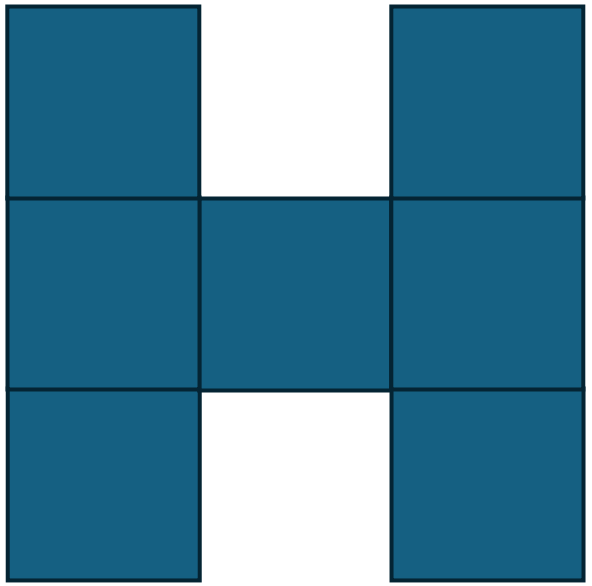

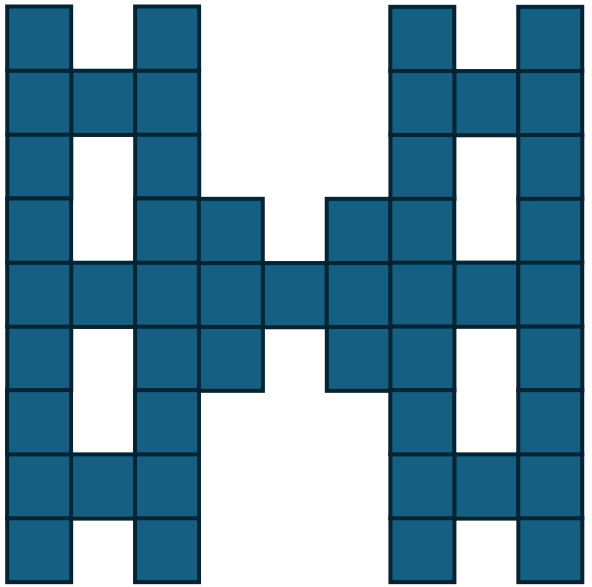

Future Work

- Does this extend to **ALL** generators?
- Does there exist more efficient methods?
  - Linear in size of generator
  - Implement a counter encoded in the fractal?
- Are there any provable lower bounds?
- Does there exist a **SINGLE** system that can strictly build any fractal with any arbitrary generator?